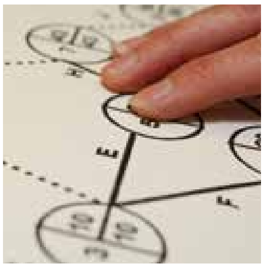
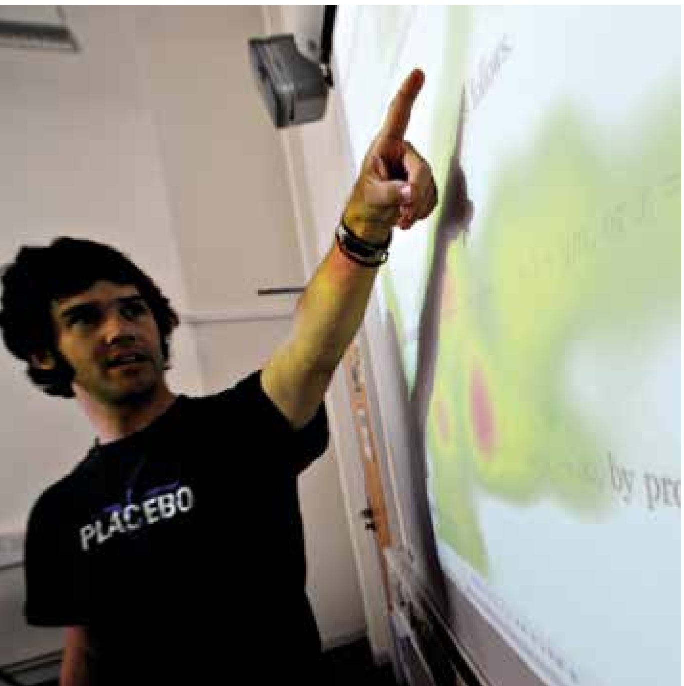


centre for excellence
sigma Σ
in mathematics & statistics support

Understanding the UK Mathematics Curriculum Pre-Higher Education

A Guide for Academic Members of Staff

2016 Edition



This guide was produced by MEI and **sigma**

MEI Innovators in Mathematics Education

Authors
Stephen Lee (Lead Author), Research and Evaluation Manager, MEI
Email: Stephen.lee@mei.org.uk

Stephen studied for a BSc (Hons) in Mathematics with Education and a PhD in Mathematics Education at Loughborough University. His thesis focussed upon Mathematics at the transition from School/College to University. He has conducted much research and has many publications in this area. In 2008 he authored an undergraduate textbook on introductory mathematics.

Published January 2016

Published by
sigma - Network for Excellence in Mathematics and Statistics Support
Mathematics Education Centre
Loughborough University
Loughborough
Leicestershire
LE11 3TU
United Kingdom

ISBN: 978-1-911217-05-3

contents

1. Background

1.1 The rationale for the document

In order to study a wide range of undergraduate programmes, including those in the Biological Sciences, Chemistry, Computer Science, Engineering, Materials Science, Mathematics and Physics, students need to have gained a mathematics qualification prior to entering into university-level study.

A considerable number of pre-higher education mathematics qualifications are available within the UK and, for those working within the Higher Education (HE) sector, it is not always clear what mathematics content, methods and processes students will have studied or indeed can be expected to know and understand as they commence their university-level programmes.

In 2010, the Maths, Stats & OR Network of the Higher Education Academy, in conjunction with the HEA Subject Centres for Bioscience, Engineering, Information and Computer Sciences, Materials and Physical Sciences commissioned Mathematics in Education and Industry (MEI) to write a mathematics guide. This was written for those within the HE sector, to outline what students with given prior qualifications in mathematics are likely to know and be able to do. Note it does not incorporate 'other' qualifications which may include elements of mathematics and/or statistics within them.

This is the 2016 edition and incorporates updates to qualifications since the previous 2010 edition was created. It has been made possible through funding from **sigma** (www.sigma-network.ac.uk).

1.2 Introduction and overview

Chapter 2 sets the scene on pre-university qualifications and study. This includes an introduction to the main qualifications, a brief historical review of major developments and an overview of where and how entrants have studied prior to starting HE.

The main content of the guide is encapsulated within Chapter 3, with information on specific qualifications and attributes of students who enter HE with them. Information about qualifications is given in the short sections; if the user just wishes to refer to a particular qualification, it should be straightforward to identify the relevant section of this chapter.

Chapter 4 provides useful sources of information. This is broken down into two parts, the first part giving links to specific references raised in the previous chapter, and the second on additional links to other documents (useful for gaining a more detailed understanding) and to relevant organisations (where information and updates can be found).

The guide concludes with appendices, including one on acronyms used and one which presents the statistics on the number of entrants to mathematics A levels over the last 20 years.

Overall this guide will give an overview of the key mathematics qualifications and offers links to further information that should aid the reader to gain an understanding of pre-university mathematics qualifications.

2. Setting the scene: Pre-higher education qualifications and study

2.1 Introduction to the main qualifications

In April 2013, the Department for Education (DfE) published a policy paper, '2010 to 2015 government policy: school and college qualifications and curriculum'. This paper outlined changes to curriculum and qualifications in order to better prepare students for life after school. The General Certificate of Secondary Education (GCSE) qualifications, which are usually taken at age 16, would be reformed and made more challenging. General Certificate of Education, Advanced level (GCE A level) qualifications, which are usually taken at age 18, would be reformed in order to ensure that they are a good preparation for HE. AS level can be taken after one year of study, as a separate qualification to A level. The new qualifications will be taught from September 2015, 2016 or 2017, depending on the subject. The new mathematics GCSEs are being taught from September 2015, for first examination in summer 2017, and the new mathematics A levels will be taught for the first time from September 2017.

The government has expressed an ambition that, by 2020, the great majority of young people will continue to study mathematics to age 18. From September 2013, students who reach age 16 without a good GCSE pass (see section 3.1.1 for details on a 'good' pass in GCSE Mathematics) in Mathematics and English, continue to work towards these qualifications.

New Core Mathematics qualifications have been introduced for students who have gained a good pass in GCSE Mathematics, but do not wish to continue to study mathematics at A level.

Apprenticeships combine paid work with on-the-job training, qualifications and progression. They include a requirement to take Mathematics and English qualifications at an appropriate level.

General qualifications in mathematics provide the evidence of attainment in mathematics that is most likely to be presented to HE admissions tutors. This guide will clarify the content, style of assessment and probable learning outcomes that may be expected in a number of general qualifications in mathematics; these are GCSE, A level, Core Maths and Free Standing Mathematics Qualifications (FSMQ). The International Baccalaureate (IB) and the Pre-University (Pre-U) will also be cited. There are a number of other qualifications available at Level 2 (i.e. GCSE) and Level 3 (i.e. AS/A level), which contain some mathematics, but it is not the intention to review these other than to list many of them here for reference – such qualifications include: Functional Skills, Quantitative Methods, Cambridge Progression, Edexcel Awards in Mathematics, Edexcel Certificate, AQA Entry Level Certificate and AQA Certificate.

2.2 Brief historical review of major developments

General qualifications in mathematics have developed in the context of widespread recent changes in expectations for learners.

The Cockcroft Report, 'Mathematics Counts' (1982) shows that in 1979 about one third of school leavers took the General Certificate of Education, Ordinary Level (GCE O level) in Mathematics with one fifth of all school leavers gaining a pass grade. The replacement of GCE O level by GCSE in 1988 may be seen as the start of a process by which these 'school leaving' qualifications could more closely reflect what the majority of 16-year olds know, understand and can do. The substantial problem solving requirement of O level Mathematics was replaced by GCSE examinations that have required candidates to show capability in handling a broad range of basic mathematics questions. The reformed GCSE Mathematics, which will be examined for the first time in 2017, has an increase in content and a greater emphasis on mathematical problem solving.

The introduction of subject cores for A level examinations in 1983, the acknowledgement in 1996 that the AS standard should be pitched according to what is likely to be achieved a year before taking A level, and the rise of modular assessment at A level since 1990 have all played significant parts in making A level Mathematics examinations much more accessible than they were between 1951 and 1983. Until 1987 results were norm referenced so that in any subject 10%

attained grade A, 15% B, 10% C, 15% D, 20% E and a further 20% were allowed an O level pass. Grading of A level in recent years has used a combination of examiner judgement and statistical information, with the process overseen by Ofqual to ensure comparability of standards between different exam boards and over time. There has been a large growth in numbers of students taking Mathematics and Further Mathematics A levels from 2005 to the time of writing.

New A levels in Mathematics and Further Mathematics are being introduced for first teaching from 2017; representatives from HE have been involved in the development of content through the A level Content Advisory Board (ALCAB). There will be an increased emphasis on mathematical modelling, reasoning and problem solving but there is no intention to make the qualifications more difficult.

A number of reports (e.g. Hillman 2014, Hodgen and Pepper 2010 (1)) have shown that fewer students in England continue with mathematics post-16 compared to many other countries. New Core Mathematics qualifications were introduced for first teaching in September 2015 (with some early adopters starting in 2014) in order to provide an alternative pathway to AS/A level Mathematics, for students who have a good pass in GCSE Mathematics, but who do not intend to progress to HE courses or employment that make significant use of mathematics.

2.3 Where and how will entrants have studied pre-higher education?

It is important to be clear that those entering degree courses come from a wide range of backgrounds and bring with them a wide range of experiences. Two overarching factors relevant to this are ***where*** and ***how*** an entrant studied previously.

This information guide is made with particular reference to those entering onto a degree from a UK background, i.e. not from overseas. With respect to the situation in England (Wales, Scotland and Northern Ireland will be dealt with in section 3.7), the major breakdown of categories of places of learning is in terms of age range, type and whether it is state-funded or independent (fee paying).

Over 90% of the secondary population attend state (government-funded) schools and, in the context of study prior to entry to higher education, establishments could include many different age ranges and have a varied focus. Age ranges for secondary study could involve 11-18, 14-19 or 16-19. The last of these could be small sixth forms attached to a school or they could be huge stand-alone Colleges of Further Education (FE) or Sixth Form Colleges. A learner may have been at the same place of study since the age of 11, or may have been at an establishment for only one or two years to complete his or her pre-HE studies.

In terms of the independent sector, it is widely expected that many of those attending such establishments will have been exposed to high quality teaching and learning resources, and although there is only a small proportion of the age cohort attending such establishments nationwide, most will go on to enter HE.

It is very difficult to describe definitively the way students will have been taught in all of these different establishments. Be it in the state or independent sector, some will have been in small classes whilst others will have been in large classes, some will have had well-qualified mathematics teachers/lecturers, others non-qualified mathematics teachers/lecturers. What is apparent, though, is that learners will enter HE with different teaching and learning experiences in relation to mathematics and will respond to the relative changes that university-level study brings in different ways. This is the case without even considering the specific subject knowledge, which will be detailed in the next chapter. Thus all students will benefit from support on how to adapt to learning the mathematical content of various different degree programmes.

3. Specific UK qualifications and attributes of students who enter higher education with them

This section describes the structure and content of specific UK mathematics qualifications and attempts to indicate the likely attributes of students who have taken them. However, the content of qualification specifications cannot be assumed to be an accurate indicator of what students will actually know and understand when they start Higher Education. This will be influenced considerably by the nature of their mathematical learning experiences and by the grades they achieved.

Several universities have used diagnostic tests to determine the mathematical knowledge, understanding and fluency of new undergraduates, and how they relate to students' qualifications at the start of their HE courses.

GCSE and A level qualifications are examined by three awarding bodies in England: Assessment and Qualifications Alliance (AQA), Edexcel and Oxford, Cambridge and RSA (OCR). They are regulated by the Office of the Qualifications and Examinations Regulator (Ofqual). GCSE and A level qualifications are also taken by students in Wales and Northern Ireland, though the arrangements for administration are different. Scotland operates a separate system of examinations.

The large majority of students entering HE have taken GCSE and A level qualifications, but several other qualifications are used as routes into HE.

3.1 General Certificate of Secondary Education

3.1.1 Overview

Students in state schools in England must follow the National Curriculum until age 16. A revised 'National Curriculum in England: mathematics programmes of study' document and also a 'Mathematics – GCSE subject content and assessment objectives' document were published in 2013.

Students will take the new GCSE Mathematics examinations for the first time in summer 2017. GCSE Mathematics assesses the mathematics National Curriculum and is usually taken by students at age 16. Although the GCSE course is often thought of as a two year course, the GCSE work in mathematics builds directly on earlier work in mathematics and so the GCSE examinations test the mathematics that students have learnt throughout secondary school (11-16), and earlier. The content of GCSE Mathematics is the same for all awarding bodies. GCSE Mathematics is a linear qualification and all of the assessment is by examination (three papers), taken at the end of the course.

Many students do not do any more mathematics after GCSE. Such students will usually not have done any mathematics for two or three years before starting their degree courses and may have limited recollection of GCSE Mathematics content and techniques.

GCSE Mathematics is available at either Foundation Tier or Higher Tier. The new GCSE Mathematics is graded on a 9 point scale, with 9 as the highest grade. In 2017, a grade 4 on the new GCSE will equate to grade C on the previous GCSE Mathematics and will be considered a pass grade (the DfE has indicated an aspiration that the 'pass' grade might be increased to grade 5 at some point in the future). Grade 7 on the new GCSE Mathematics equates to a grade A on the previous GCSE Mathematics. Grades 8 and 9 equate to the previous A* grade, with grade 9 indicating really exceptional performance. Grades 1 to 5 are available at Foundation Tier and grades 4 to 9 are available at Higher Tier. If a student is a small number of marks below the grade 4 boundary on the Higher Tier, that student may be awarded a grade 3, rather than U (unclassified). In 2015 62.4% of students taking GCSE Mathematics achieve grade C or above.

Students entering Mathematics GCSE at Foundation Tier will not have studied as much mathematics as students taking Higher Tier. However, the grade boundaries for grades 4 and 5 at Foundation Tier will be higher than for Higher Tier, so Foundation Tier students with grades 4 or 5 have shown a good understanding of the mathematics they have studied.

3.1.2 Subject knowledge and skills

Content of the GCSEs in Mathematics taken from summer 2017 will be similar to that of the previous GCSE Mathematics, but they are intended to be more mathematically demanding and do contain a little more material, most notably inverse and composite functions and areas under graphs at Higher Tier and expanding, factorising and solving quadratics, plus simple trigonometry at Foundation Tier. The examinations will put more emphasis on reasoning, problem solving and functionality in mathematics.

Students who have not gone beyond the content of Foundation Tier GCSE will not have met some topics which students taking Higher Tier will have encountered. The list below covers the main curriculum content that Foundation Tier students for the new GCSE Mathematics will not normally be taught. This list is not exhaustive and the reader would be advised to satisfy themselves of the finer detail by reviewing an Awarding Body's specification, see for example:
www.ocr.org.uk/Images/168982-specification-gcse-mathematics-j560.pdf

- simplify surd expressions involving square roots (e.g. $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$) and rationalise denominators
- change recurring decimals into their corresponding fractions and vice versa
- simplify and manipulate algebraic fractions
- factorise quadratic expressions of the form $ax^2 + bx + c$
- interpret the reverse process of a function as the 'inverse function'; interpret the succession of two functions as a 'composite function'
- analyse quadratic functions by completing the square
- be familiar with the graphs of exponential or trigonometric functions
- calculate or estimate gradients of graphs and areas under graphs and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts
- recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point
- find approximate solutions to equations numerically using iteration
- using the quadratic formula
- trigonometry in 3D
- finding the *nth* term of quadratic sequences
- interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of average and instantaneous rate of change (gradients of chords and tangents) in numerical, algebraic and graphical contexts
- describe the changes and invariance achieved by combinations of rotations, reflections and translations
- apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results
- know and apply the sine rule, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, and cosine rule, $a^2 = b^2 + c^2 - 2bc\cos A$, to find unknown lengths and angles
- know and apply $\text{Area} = \frac{1}{2}ab\sin C$ to calculate the area, sides or angles of any triangle.
- use vectors to construct geometric arguments and proofs
- calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams
- construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use

Students who have been entered for Higher Tier Mathematics and achieved grades 4, 5 or 6 are likely to have an incomplete understanding of items from the list above. Those with a grade 4 may not be fluent with basic algebra.

3.1.3 International General Certificate of Secondary Education

The International General Certificate of Secondary Education (iGCSE) was originally designed for international schools but is now taken by students in some independent and state schools in the UK. There are Cambridge iGCSEs and Edexcel iGCSEs.   The standard and content are similar to GCSE but students may have studied some additional topics, such as an introduction to calculus or matrices.

3.2 Advanced Subsidiary and Advanced Levels

3.2.1 Overview

Advanced Subsidiary and Advanced Level qualifications in Mathematics and Further Mathematics taken between years 2005 and 2018 follow a modular structure, as outlined below. Changes are planned for teaching from 2017, which will affect students taking A level Mathematics in 2019; A level Mathematics will then be a linear qualification, with all the assessment taken at the end of the course.

AS qualifications are typically taken after one year of study; the content is a subset of the A level qualification which typically takes two years of study.

3.2.2 Modular mathematics A levels (2005-2018)

Modular AS level Mathematics, Further Mathematics and Statistics each consist of three modules (also called ‘units’). A level in each of these subjects consists of six modules, which include the three AS modules. Students who have A level will also have studied the AS content but they may not have requested the certification for the AS separately so it might not appear on their certificate. The modules in these subjects are all of equal size and carry the same number of marks.

The raw marks on each module are converted to Uniform Marks (UMS), to allow for slight differences in difficulty of examinations from year to year; the overall grade is decided by the total Uniform Mark gained on all modules. Students could resit individual modules to improve their marks.

The modules which are available in the MEI Mathematics and Further Mathematics A levels are shown in Figure 1. Similar structures apply for the other Mathematics and Further Mathematics A levels.

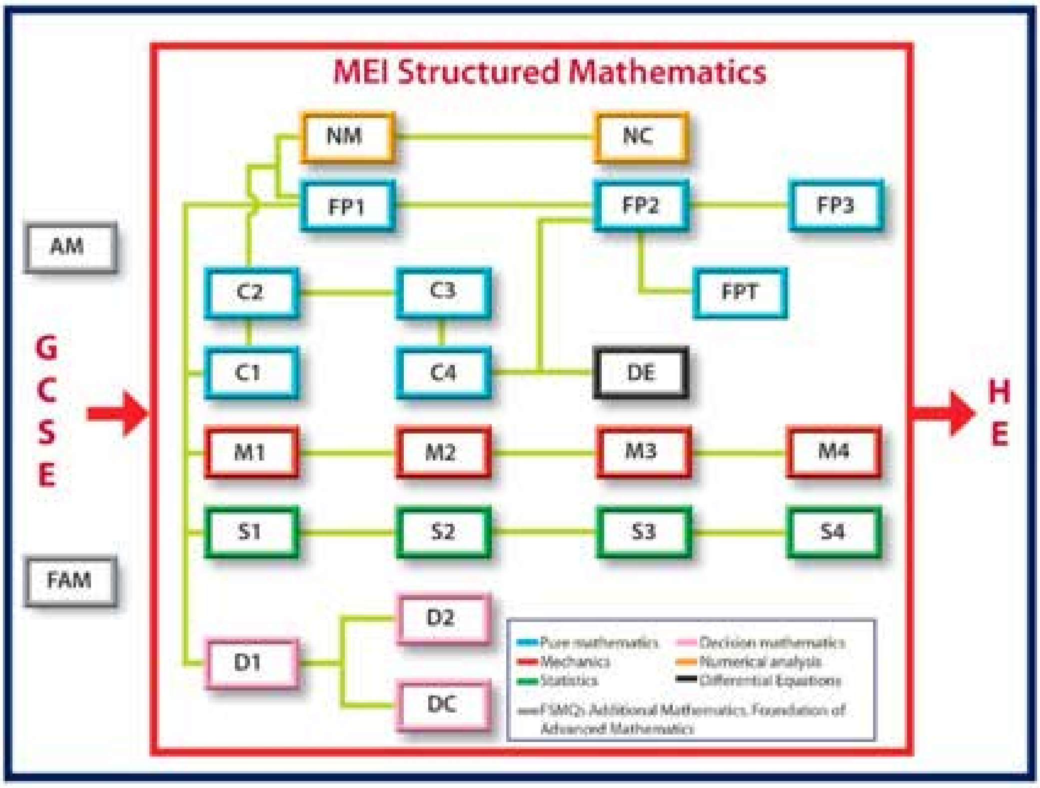


Figure 1

(Figure 1 notes – AM is Additional Mathematics, FAM is Foundations of Advanced Mathematics, NM is Numerical Methods, NC is Numerical Computation, FP is Further Pure Mathematics, FPT is Further Pure Mathematics with Technology, C is Core Mathematics, DE is Differential Equations, M is Mechanics, S is Statistics, D is Decision Mathematics, DC is Decision Mathematics Computation)

Modular AS Mathematics consists of the compulsory modules C1 and C2 and an applied module; this could be in mechanics, statistics or decision mathematics. A level Mathematics has three further modules; the compulsory modules C3 and C4 and another applied module. The two applied modules in A level Mathematics can be from the same area of applied mathematics or from different areas. The content of C1 and C2 together (AS) is nationally specified; likewise for C3 and C4 (A2). The content of applied modules varies between different exam awarding bodies. The national core can be found in the criteria for A level Mathematics see (2). This document also details what students who achieve grade A, C or E can typically do; this only gives a general idea as grades are based on total marks achieved rather than on these criteria, so strengths in some areas may balance out relative weaknesses in others. Students with the full A level will have a broader knowledge of the AS core content than the A2 content because they have further developed their understanding in the second year.

Typically students complete AS levels after one year, either stopping their study of mathematics at this point or going on to complete the full A level in a further year. Modular Further Mathematics is only taken by students who are also taking Mathematics. They take three further modules for AS, including one compulsory module, Further Pure 1. To complete the A level in Further Mathematics, students take at least one more pure module and two other modules. Students taking A level Mathematics and A level Further Mathematics take 12 different modules and students taking A level Mathematics and AS level Further Mathematics take 9 modules.

The optional modules in AS and A level Further Mathematics can be drawn from either pure mathematics or applied mathematics. Applied modules are in suites for the three strands of applications: mechanics, statistics and decision mathematics. Mechanics 1 and Mechanics 2 can be taken in either A level Mathematics or A level Further Mathematics but Mechanics 3, 4 (and higher) are only available to students taking Further Mathematics. This is similar for modules in statistics.

For students who take both Mathematics and Further Mathematics AS and/or A level under the modular system, the Mathematics qualification consists of the compulsory core modules (C1 to 4) and a valid combination of applied modules. The remaining modules make up the Further Mathematics qualification. If there is more than one possible valid combination of applied modules to award A level Mathematics, the combination of modules making up the separate AS or A levels is automatically decided by the examination awarding body's computer in order to maximise the pair of grades students received for Mathematics and Further Mathematics. The rules for aggregation and certification can be seen in (3).

A table of the numbers of students who have taken A level Mathematics (and Further Mathematics) can be seen in appendix 5.2.

A small number of students take 15 modules to gain A levels in Mathematics and Further Mathematics and AS Further Mathematics (Additional) and some take 18 modules for A levels in Mathematics, Further Mathematics and Further Mathematics (Additional).

3.2.3 Linear mathematics A levels (2018 onwards)

Linear AS and A levels in Mathematics and Further Mathematics are being introduced for first teaching from September 2017. All the assessment takes place at the end of the course. It is still possible for students to take AS level but the marks from this will no longer contribute to the overall marks for A level.

It is intended that the reformed qualifications will place a higher emphasis on mathematical problem solving, modelling and reasoning.

For A level Mathematics, the content is ***100% specified at national level***. It includes some mechanics and some statistics. Neither schools nor students will have any choice about the content of AS or A level Mathematics. The content of AS and A level Mathematics can be found on **www.gov.uk/government/publications/gce-as-and-a-level-mathematics.**

Half the content of A level Further Mathematics is specified at national level. Different options for the remaining 50% may be possible. Further Mathematics under the linear system is a separate qualification to Mathematics and builds on the content of the A level.

At the time of publication (January 2016) the specifications detailing the exact content of awarding bodies' 2017 linear mathematics qualifications have not yet been agreed with Ofqual. Noting that A level Mathematics has 100% specified content, the differences will be in the Further Mathematics content. When these become available a summary will be available on the Further Mathematics Support Programme (FMSP) website at: **www.furthermaths.org.uk/2017-fm**

3.2.4 Subject knowledge and skills

The vast majority of A level Mathematics students will be taught in schools and colleges and so will not be used to studying mathematics independently. Most A level Mathematics examination questions are structured; however, there is an intention to increase the amount of problem solving in the linear mathematics A levels for teaching from 2017. Past papers and specimen papers are available on awarding bodies' websites (see section 4.2.2) and give an idea of what students are expected to be able to do. Students who have taken both Mathematics and Further Mathematics will have greater fluency in the subject due to the additional amount of time they have spent on it. A document giving an overview of the content studied in modular mathematics A levels can be seen in (4); this is also included in appendix 5.3.

Grades available at AS and A level are A to E and U; grades achieved on individual modules are available to universities through UCAS, as well as the result for the whole qualification. From summer 2010, grade A* has been available for the full A level (but not for AS) with around 1 in 6 obtaining the grade. For modular A level Mathematics, a total of 180 UMS marks (out of 200) are needed on the two compulsory A2 modules (C3 and C4). For modular A level Further Mathematics, a total of at least 270 (out of 300) is needed on the best three A2 modules. For the linear qualifications, it is proposed that the proportion of A* grades is largely determined by the cohort's prior attainment at GCSE. Students with A* will have shown the ability to work accurately under pressure and, it is hoped, for the new mathematics A levels, will have demonstrated the ability to solve unstructured problems that require a high level of mathematical reasoning and problem solving.

A small number of students (totalling fewer than 500) take AS or A level Pure Mathematics under the modular system; these qualifications will no longer be available in the linear system. A level Pure Mathematics consists of the four compulsory core modules from A level Mathematics together with two Further Mathematics Pure Modules; it cannot be taken with Mathematics or Further Mathematics AS or A level.

Some students (totalling around 2000) take AS or A level Statistics. This is a separate qualification from Mathematics and Further Mathematics and the focus is more on the use of statistics whereas the statistics components in the mathematics suite are more mathematical. The content of AS or A level Statistics would provide very useful background for students going on to study Business, Biology, Psychology or Social Sciences at Higher Education level.

Students who have completed their mathematical studies a year, or more, before starting Higher Education may need some support with revision to regain the fluency they had when they sat their examinations.

3.3 Advanced Extension Award and Sixth Term Examination Paper

The Advanced Extension Award (AEA) is based on A level Mathematics core content and is designed to challenge the highest attaining students. Due to the small number of candidates, the examination awarding bodies divided the subjects offered between them; Edexcel offered the Mathematics AEA. AEAs in other subjects existed until August 2009, at which point they were withdrawn as it was felt that the new A* grade in A levels would overlap with the purpose of the AEA. However, Mathematics AEA was the only AEA that remained as it is felt that it is still meeting a need. At present it is intended that the Mathematics AEA will continue until at least 2017, when the revised A levels will begin.

Students who are successful in AEA or STEP will have demonstrated the ability to think for themselves, persist with a problem and present structured mathematical arguments.

The Mathematics AEA is assessed by a 3 hour paper of pure mathematics questions with no calculator allowed. Grades available are Distinction and Merit. Although candidates do not have to learn any additional content for AEA, they do need to become familiar with a different style of question and to present clearly structured mathematical arguments.

The Sixth Term Examination Paper (STEP) is a university admissions test originally used only for entrance to Cambridge, but it is now also used by some other universities. It is administered by the examination awarding body Cambridge Assessment. There are three mathematics papers I, II and III and candidates usually take two of them. There are three mathematics papers (I, II and III) and candidates usually take two of them. Papers I and II are based on A level Mathematics and paper III is based on A level Further Mathematics. Questions may include some content that is not in the A level syllabus, but candidates are not expected to learn extra content for the examination. Each 3 hour paper has 13 questions (8 pure mathematics, 3 mechanics and 2 statistics) of which students are expected to answer 6. No calculator is allowed for any of the papers. Grades available are S – Outstanding, 1 – Very Good, 2 – Good, 3 – Satisfactory, U – Unclassified. Usually a candidate will be awarded a grade 1 for a paper if they answer 4 (out of 6) questions well.

3.4 Core Mathematics Qualifications

In order to meet the national ambition of more students continuing with mathematics until age 18, new Core Mathematics qualifications have been developed. They are intended for students who have been successful at GCSE but who do not find AS or A level Mathematics meets their needs. The qualifications are at the same level and are of the same size as AS Mathematics, but have a different focus.

Core Mathematics qualifications do not have "Core Mathematics" in their title; they were recognised as Core Mathematics qualifications after their development because they met the following characteristics.

- They develop and extend the mathematics which students have met at GCSE.
- They foster the ability to think mathematically and to select and apply appropriate techniques in a variety of situations which could be encountered in future life, work and study.
- They build skills in mathematical thinking, reasoning and communication.

At least 20% of the mathematical content of Core Mathematics qualifications must be beyond Higher Tier GCSE Mathematics; they should provide a sound basis for the mathematical demands that students will face at university and within employment across a broad range of academic, professional and technical fields. The detailed criteria that qualifications must meet in order to be counted as Core Mathematics qualifications can be found in the Technical Guidance for Core Mathematics qualifications (5).

Core Mathematics qualifications became available for first teaching from 2015 with first assessment in 2017; however, students from some 'early adopter' schools and colleges will take their qualifications in summer 2016.

The assessment of Core Mathematics takes place at the end of the course; at least 80% of the assessment must be through externally set examination. At the time of writing, six Core Mathematics qualifications are available; these are listed on the Core Mathematics Support Programme website (5). They are all graded from A to E and five of the six are assessed by examination only.

In addition to the specific aims and content of Core Mathematics qualifications, which have a strong focus on mathematical problem solving, reasoning and communication, the consolidation and extension of GCSE skills which they foster will prepare students better for the mathematical needs of future work and study than if they had stopped studying mathematics at age 16.

3.5 Free Standing Mathematics Qualifications

FSMQs were first developed in the late 1990s. The initial motivation was to support vocational qualifications, e.g. General National Vocational Qualifications (GNVQ's), but it was also recognised that they could provide useful courses for other students as well. Uptake of the original FSMQs was not high and only AQA now offers them in their original form. They are tightly focused qualifications in 'Working with Algebraic and Graphical Techniques', 'Using and Applying Statistics', 'Modelling with Calculus' and 'Using and Applying Decision Mathematics'; they compensate for their narrow focus by requiring quite deep coverage.

OCR offer two FSMQs which cover mathematics more broadly, but in less depth. Foundations of Advanced Mathematics (FAM) is a level 2 qualification that is designed to help bridge the gap between GCSE and AS Mathematics for students with a C/B grade Mathematics GCSE. Additional Mathematics is a level 3 qualification, comparable in difficulty to AS Mathematics, which is aimed at high attaining GCSE students and designed to be taken alongside GCSE.

The AQA qualifications are likely to be used to support study of a range of courses in subjects other than mathematics. The OCR qualifications are more likely to be used to demonstrate achievement of a milestone in a learner's mathematical development.

All FSMQ qualifications are similar in size, rated at 60 guided learning hours, the same size as a single unit of a modular A level.

Level 3 units are awarded Universities and Colleges Admissions Service (UCAS) points.

The AQA FSMQs share a single assessment model. Candidates must produce a coursework portfolio worth 50% of the credit and sit a written examination for the remaining credit. The OCR FSMQs use slightly different assessment approaches, but both are assessed by written examination only.

Students who have achieved success in these qualifications are likely to share the broad capabilities of students achieving other mathematics qualifications at Levels 1, 2 and 3. However, students who have taken the AQA FSMQs will have demonstrated the ability to appreciate real world use and application of mathematics; they will also have engaged with completing a substantial coursework project. Students who have achieved success with OCR Additional Mathematics are likely to have shown an excellent grasp of basic advanced topics, which should be valued more highly if the qualification was taken pre-16. Students who have been successful in FAM will have studied a broader range of mathematics and are therefore more likely to be able to meet the demands of mathematics post-GCSE, particularly in algebra and trigonometry.

3.5.1 Advanced Subsidiary Use of Mathematics

Students who take level 3 FSMQs with AQA have an opportunity to use these as part of an AS qualification called AS Use of Mathematics. To achieve this requires two of the level 3 FSMQs, one of which must be 'Working with algebraic and graphical techniques', together with a terminal unit, 'Applying Mathematics'. Coinciding with the introduction of the new mathematics A levels the AS Use of Mathematics qualification will have its last examinations in summer 2016, with students able to certificate for the qualification until summer 2017. A piloted full A level in Use of Mathematics will also to be withdrawn.

3.6 Other Qualifications

3.6.1 International Baccalaureate

The International Baccalaureate (IB) Diploma Programme is recognised as a challenging two-year course for students aged 16-19. In the UK in 2015, approximately 125 schools offered the qualification.

The course consists of a core, made up of three separate parts, and then six subject groups, from which students select six subjects. Normally three subjects are studied at standard level (representing 150 teaching hours and broadly equivalent to AS levels) and three are studied at higher level (representing 240 teaching hours and broadly equivalent to A levels).

Four courses are available in mathematics: mathematics studies or mathematics at standard level, and mathematics or further mathematics at higher level.

These courses aim to enable students to develop mathematical knowledge, concepts and principles; develop logical, critical and creative thinking and employ and refine their powers of abstraction and generalization.

All students who study the IB are required to study one of the mathematics qualifications. The full specification for mathematics and further mathematics at higher level can be seen in (6).

3.6.2 Cambridge Pre-U

The Pre-U is a recent qualification that had its first entry certifications in 2010. It has a linear structure with examinations at the end of the two year course.

Students study at least three Principal Cambridge Pre-U subjects from a choice of 25, see (7). They can also complete an Independent Research Report and a Global Perspectives portfolio. Mathematics and Further Mathematics are individual subjects in this list of 25.

It was indicated in Gill (2015) 'Uptake of level 3 qualifications in English schools 2014' that IB and Pre-U each had an uptake of around 3000 students.

3.6.3 Access Courses

Access to HE courses are specifically designed for people who left school without the usual qualifications but who would like to take a degree level course. Although there is a dedicated website where some information can be sought, see (8), the content of courses varies. It is advisable for institutions to review students entering with such qualifications on an individual basis to determine their prior mathematical experience.

3.6.4 Foundation Courses

These courses are run by some universities to offer students who are unable to apply directly to a degree programme an opportunity to study in an area or subject of their interest and progress to a level where, on successful completion of the course, they can enter onto an undergraduate course.

Although it is perhaps usual for a learner on such a course to progress onto a subsequent course at the institution at which they had studied their Foundation course, this need not necessarily be the case. However, these courses are institution specific and so it would again be advisable for institutions to review on a case-by-case basis what such a course entailed and thus what can be expected of students who have completed it.

3.6.5 Vocational Qualifications

There are numerous other pre-university qualifications available that fall under a 'vocational' classification. Such courses can be suitable for students intending to undertake further study at university. Many of these qualifications also have options available that are equivalent to first degree and postgraduate level. The DfE has a useful webpage that lists comparable qualifications at the various levels of study, see (9).

One example of such a qualification is BTECs. BTEC Firsts are available up to equivalent to GCSE level, with BTEC Nationals available from equivalent to A level standard. Overall, there are more than 2000 BTEC qualifications available across 16 sectors, i.e. business, construction, engineering, etc, so it's not possible to succinctly list the mathematics within them here. Readers may wish to review specific courses available from an Awarding Body, see (10).

3.7 Wales, Scotland and Northern Ireland

In ***Wales*** there is much overlap with England in the mixture of post-16 courses and the differing establishments available. However, most students sit examinations with the examination awarding body, the Welsh Joint Education Committee (WJEC). Some students educated in Wales are taught using English specifications and take examinations with the English awarding bodies. In September 2015 Qualifications Wales was established as the regulator of qualifications in Wales. WJEC is to be rebranded as Eduqas for new qualifications being taught from 2015 (the legacy specifications will remain branded as WJEC to differentiate them).

Over the past five years, 2010-2015, the number studying A level Mathematics has steadily increased from circa 3300 to circa 3700. A level Further Mathematics numbers are still comparatively low at around 500 in 2015 (but this has doubled from 240 in 2010).

In ***Scotland***, a Curriculum for Excellence (CfE) was introduced for 3-18 year olds in 2010. This aimed to provide a coherent, flexible and enriched curriculum. The 'senior phase' of education takes place between the ages of 16 to 18. Prior to CfE: at age 16 students took Standard Grade examinations (roughly equivalent to the English GCSEs), at age 17 they took Highers (equivalent to AS levels) and then at 18 they took Advanced Highers (equivalent to A levels). From 2013 the Standard Grade exam was replaced with new 'Nationals' (National 5 being the highest level qualification), new Highers were introduced in 2014 and new Advanced Highers in 2015. It is worth noting that although many students do stay on to study Advanced Highers, students going on to study at a Scottish university can gain entrance having studied only Highers.

Pre-2013 qualifications: Grades for Standard Grades were on a scale from 1 to 7, where 7 was the lowest and awarded if the course was completed but the examination not passed. There were 3 levels of difficulty of mathematics papers: Foundation (grades 5 and 6), General (grades 3 and 4) and Credit (grades 1 and 2). Each level had 2 papers, only one of which allowed a calculator. Both papers assessed knowledge, understanding, reasoning and enquiring skills.

There were also two Intermediate Mathematics courses, Intermediate 1 and Intermediate 2, which could be studied instead of Standard Grades, or taken alongside Highers. The Intermediate 1 Course was equivalent to the knowledge and skills developed in Standard Grade Mathematics at Foundation level and the Intermediate 2 Course was designed to be equivalent to the knowledge and skills developed in Standard Grade Mathematics at General level. Their structure was like that of Highers, i.e. a one-year course split into units.

Post-2013 qualifications: Grading for the National 5, Highers and Advanced Highers in their new specifications are on a scale of A to D and 'no award', rather than on the previous numerical scale of 1-7. Further details on the mathematics qualifications taken in the senior years can be seen on the Scottish Qualifications Authority's (SQA) website (11) – SQA being Scotland's only examination awarding body.

In ***Northern Ireland*** there are AS and A levels in Mathematics and Further Mathematics. Over the past five years, 2010-2015, the number studying A level Mathematics has increased from circa 3000 to circa 3300. A level Further Mathematics numbers are still very low at around 180 in 2015 (but this has seen a 40% increase from 130 entries in 2010). The examinations are provided by the Council for the Curriculum, Examinations and Assessments (CCEA), which is also the regulatory body.

4. Useful Sources of information

It is clear that there is much more to be said than can be included in a concise guide such as this. Links to relevant sources of information are given here. Most of these will consist of links to build upon information given earlier, but there are also some links to other material which would be useful for gaining a deeper understanding.

4.1 References made in this guide

(1) Hillman, J. (2014) Mathematics After 16: The State of Play, Challenges & Ways Ahead. London: Nuffield Foundation.
Hodgen, J & Pepper, D (2010) Is the UK an outlier? An international comparison of upper secondary mathematics education. London: Nuffield Foundation

(2) The A level and AS qualifications requirements can be seen at:
www.gov.uk/government/collections/a-level-and-as-level-qualifications-requirements

(3) The rules for A level aggregation and certification can be seen at:
www.jcq.org.uk/exams-office/entries/gce-maths-information

(4) Overview of content in modular mathematics A levels (2008-2018)
www.mei.org.uk/files/Maths_Alevel_Content_09.pdf

(5) Core Mathematics qualifications: www.core-maths.org/information/awarding-organisations
Core Mathematics technical guidance: www.gov.uk/government/publications/core-maths-qualifications-technical-guidance

(6) Full specification for IB mathematics at higher level can be seen at:
www.ncbis.net/Editor/EditorImages/Math%20HL.pdf
Full specification for IB further mathematics at higher level can be seen at:
http://bit.ly/IBFurtherMathsSL

(7) Information on the subjects available in the Pre-U can be seen at:
www.cie.org.uk/programmes-and-qualifications/cambridge-advanced/cambridge-pre-u

(8) Access courses website: www.accesstohe.ac.uk

(9) Comparable qualifications, DfE:
www.gov.uk/what-different-qualification-levels-mean/compare-different-qualification-levels

(10) BTEC qualifications from one Awarding Body
http://qualifications.pearson.com/en/qualifications/btec-nationals.html

(11) SQA Mathematics qualifications: www.sqa.org.uk/sqa/45750.html

4.2 Additional references

4.2.1 Documents/information

Cockcroft report (1982): www.educationengland.org.uk/documents/cockcroft/cockcroft1982.html
(A report of the Committee of Inquiry into the teaching of Mathematics in Schools)

Evaluation of participation in A level mathematics: Final report, QCA (2007)
www.nottingham.ac.uk/emp/public/publications/qca maths gce eval report.pdf

Ideas from Mathematics Education - An Introduction for Mathematicians
Authored by Lara Alcock & Adrian Simpson. ISBN 978-0-9555914-3-3
https://dspace.lboro.ac.uk/dspace-jspui/bitstream/2134/8846/1/Alcock_Simpson_book.pdf

IB www.ibo.org/who and www.ibo.org/diploma/curriculum

iGCSE: www.cie.org.uk/qualifications/academic/middlesec/igcse/overview

Made to Measure, Ofsted (2012)
www.gov.uk/government/uploads/system/uploads/attachment_data/file/417446/Mathematics_made_to_measure.pdf

Mathematical Association spreadsheet on the changes in school mathematics 1999-2016
www.m-a.org.uk/resources/TheChangingShapeOfTheCurriculum.xls

Mathematics: understanding the score, OFSTED (2008) http://stem.org.uk/rx8ok

Mathematics careers website www.mathscareers.org.uk

Report on Mechanics at the transition from School to University:
www.mei.org.uk/files/pdf/NewtonMechReportFinal.pdf

STEP www.admissionstestingservice.org/for-test-takers/step/about-step/

What different qualifications mean www.gov.uk/what-different-qualification-levels-mean

4.2.2 Organisations

A list of organisations which serve to provide additional information on mathematics and/or education are included here (note that not all have previously been mentioned explicitly in this guide).

- **ACME** www.acme-uk.org
- **AQA** http://web.aqa.org.uk/qual/gceasa/mathematics.php
- **ATM** www.atm.org.uk
- **BIS** www.gov.uk/government/organisations/department-for-business-innovation-skills
- **CCEA** www.rewardinglearning.org.uk/microsites/mathematics/gce/index.asp
- **DfE** www.education.gov.uk
- **Edexcel** www.edexcel.com/quals/gce/gce08/maths/Pages/default.aspx
- **Eduqas** www.eduqas.co.uk
- **FMSP** www.furthermaths.org.uk
- **JCQ** www.jcq.org.uk
- **MA** www.m-a.org.uk/jsp/index.jsp
- **MEI** www.mei.org.uk
- **OCR** www.ocr.org.uk/qualifications/subjects/mathematics/index.aspx
- **OFQUAL** www.ofqual.gov.uk
- **QCDA** www.qcda.gov.uk
- **SQA** www.sqa.org.uk
- **SCQF** www.scqf.org.uk
- **WJEC** www.wjec.co.uk

5. Appendices

5.1 Acronyms used in this guide (including appendices)

ACME Advisory Committee on Mathematics Education
AEA Advanced Extension Award
ALCAB A level Content Advisory Board
AS/A level Advanced Subsidiary/Advanced level
AQA Assessment and Qualifications Alliance
ATM Association of Teachers of Mathematics
BIS (department for) Business, Innovation and Skills
CCEA Council for the Curriculum, Examinations and Assessments
CSE Certificate of Secondary Education
DfE Department for Education
DfES Department for Education and Skills
FAM Foundations of Advanced Mathematics
FE Further Education
FM Further Mathematics
FMSQ Free Standing Mathematics Qualifications
GAIM Graded Assessment in Mathematics
GCSE General Certificate of Secondary Education
GCE O Level General Certificate of Education Ordinary Level
GNVQ General National Vocational Qualification
HE Higher Education
IB International Baccalaureate
iGCSE International GCSE
JCQ Joint Council for Qualifications
MA Mathematical Association
MEI Mathematics in Education and Industry
OCR Oxford Cambridge and RSA Examinations
OFQUAL Office of the Qualifications and Examinations Regulator (in England)
SCQF Scottish Credit and Qualifications Framework
SCUE Standing Conference on University Entrance
SQA Scottish Qualifications Authority
STEM Science, Technology, Engineering and Mathematics
STEP Sixth Term Examination Paper
UCAS Universities and Colleges Admissions Service
UMS Uniform Mark Scale
WJEC Welsh Joint Education Committee (Eduqas is the rebranded qualifications from WJEC from 2015)

5.2 A level Mathematics Numbers 1989 – 2015 (Source JCQ)

Year	Mathematics* entries (FM excl)	FM* entries	Total Mathematics entries (FM incl)	FM as % of Mathematics	Total A level entries	Mathematics as % of total entries (FM incl)
1989			84 744		661 591	12.8
1990			79 747		684 117	11.7
1991			74 972		699 041	10.7
1992			72 384		721 024	9.9
1993			66 340		734 081	9.0
1994			64 919		732 974	8.9
1995			62 188		725 992	8.6
1996			67 442		739 163	9.1
1997			68 880		777 710	8.9
1998			70 554		794 262	8.9
1999			69 945		783 692	8.9
2000			67 036		771 809	8.7
2001			66 247		748 866	8.8
2002			53 940		701 380	7.7
2003	50 602	5315	55 917	10.5	750 537	7.5
2004	52 788	5720	58 508	10.8	766 247	7.6
2005	52 897	5933	58 830	11.2	783 878	7.5
2006	55 982	7270	63 252	13.0	805 698	7.9
2007	60 093	7872	67 965	13.1	805 657	8.4
2008	64 593	9091	73 684	14.1	827 737	8.9
2009	72 475	10 473	82 948	14.5	846 977	9.8
2010	77 001	11 682	88 683	15.2	853 933	10.4
2011	82 995	12 287	95 282	14.8	867 317	11.0
2012	85 714	13 223	98 937	15.4	861 819	11.5
2013	88 060	13 821	101 881	15.7	850 752	12.0
2014	88 816	14 028	102 844	15.8	833 807	12.3
2015	92 711	14 993	107 704	16.2	850 749	12.7

* Note that prior to 2003 JCQ did not report Mathematics and Further Mathematics numbers separately.

5.3 What mathematics do students study in A level Mathematics courses (2005-2018)?

Since the structure of A level Mathematics and Further Mathematics was changed in September 2004, students with a single A level in Mathematics will have studied only two applied modules (in addition to the four core modules, Core 1 to Core 4, which cover the compulsory 'pure' content of the A level). Prior to 2004 students needed to study three pure units.

Possible combinations of modules studied for A level Mathematics are:

Core 1, Core 2, Core 3, Core 4 + one of the combinations of 2 applied modules shown below					
Statistics 1 Mechanics 1	Mechanics 1 Decision 1	Decision 1 Statistics 1	Statistics 1 Statistics 2	Decision 1 Decision 2	Mechanics 1 Mechanics 2

There are no prescribed applied modules that are required to be studied, hence students could study any one of these combinations in order to gain an A level in Mathematics.

The following pages summarise the approximate content of the four core modules in A level Mathematics, in AS/A level Further Mathematics and in the first two modules of each applied strand. However, there are differences between the content of such modules for the different A level specifications (and additionally a few other applied modules may be available from specific boards, e.g. Numerical Methods by MEI).

Those students who have studied an AS or A level in Further Mathematics will have had the opportunity to study more applied mathematics modules than those with just the single A level Mathematics. This highlights the worth of the Further Mathematics qualification for those students who wish to study for mathematics-related degrees at university. Please see **www.furthermaths.org.uk** to find out more about Further Mathematics.

There is considerable scope for MEI to work with universities to help support the learning and teaching of undergraduates, for both first year Mathematics courses and for mathematics service courses in mathematics-related degrees. For more information, please see **www.mei.org.uk** and navigate to the 'Universities' menu.

Core Mathematics

Topics that students will have met by studying the four Core modules within A level Mathematics include:

Algebra

- ✓ Simultaneous equations
- ✓ Solving quadratics, completion of square
- ✓ Surds/indices
- ✓ Logarithms
- ✓ Inequalities (only involving linear and quadratic expressions, and the modulus function)
- ✓ Polynomials (factor/remainder theorems)
- ✓ Binomial expansion
- ✓ Partial Fractions

Exponential and Log

- ✓ Their graphs
- ✓ Standard properties
- ✓ Use in solving equations

Coordinate Geometry

- ✓ Equations of straight lines, gradient
- ✓ Parallel and perpendicular lines
- ✓ Equation of a circle
- ✓ Circle theorems

Parametric Equations

- ✓ Finding gradients
- ✓ Conversion from cartesian to parametric equations

Vectors

- ✓ Scalar product
- ✓ Equations of lines
- ✓ Intersection of lines

Numerical Methods

- ✓ Roots by sign change
- ✓ Fixed point iteration

Trigonometry

- ✓ Sine rule, cosine rule
- ✓ Radians, arc length, sector area
- ✓ Exact values of sin, cos, tan of standard angles
- ✓ Sec, cosec, cot, arcsin, arccos, arctan
- ✓ Compound/double angle formulae

Sequences and Series

- ✓ Arithmetic/geometric sequences/series
- ✓ Sigma notation
- ✓ Sequences defined recursively

Curve Sketching

- ✓ Graphs of quadratics, polynomials (from the factorised form)
- ✓ Relationships between graphs of $y = f(x)$, $y = f(x + a)$, $y = f(ax)$

Functions

- ✓ Composition
- ✓ Inverses, calculating inverses
- ✓ Even, odd, periodic functions
- ✓ Modulus function
- ✓ Inverse trig functions

Calculus

- ✓ Differentiation of powers of x, e^x, ln x, sin x, cos x, tan x
- ✓ Product rule, quotient rule
- ✓ Chain rule
- ✓ Integration by inspection
- ✓ Integration by substitution (simple cases only)
- ✓ Integration by parts
- ✓ Differential equations (to include only variables which are separable)
- ✓ Implicit differentiation
- ✓ Volumes of revolution

Topics that students will **NOT** have met unless they have done AS/ A level Further Mathematics include:

Complex Numbers

- ✓ **Definitions, basic arithmetic**
- ✓ **Argand diagram**
- ✓ **Polynomial equations with complex roots**
- ✓ **Polar form***
- ✓ **De Moivre's theorem***
- ✓ **Exponential notation***
- ✓ **n^{th} roots of complex numbers***

Proof

- ✓ **Language with proof**
- ✓ **Proof by induction**
- ✓ **Proving hyperbolic trig identities**
- ✓ **Uniqueness of inverse matrix**

Coordinate Systems

- ✓ **Polar coordinates**
- ✓ **Intrinsic coordinates***

Calculus

- ✓ **Using inverse trig functions, hyperbolic trig functions**
- ✓ **More advanced substitution**
- ✓ **Integrating factor***

Matrices

- ✓ **Definitions, basic arithmetic**
- ✓ **Matrices as linear transformation, composition**
- ✓ **Determinant, inverse**
- ✓ **Use in solving linear simultaneous equations, equations of planes and geometric interpretation**
- ✓ **Characteristic polynomial***
- ✓ **Eigenvalues and eigenvectors***

Curve Sketching

- ✓ **Graphs of rational functions**
- ✓ **Conic sections***

Series

- ✓ **Telescoping**
- ✓ **Limits of series (beyond GPs)**
- ✓ **Maclaurin/taylor series and approximations**

Hyperbolic Trig Functions

- ✓ **Definition**
- ✓ **Identities**
- ✓ **Their use in calculus**

(*topics which are dependent on the material in the full A level Further Mathematics having been studied, as opposed to AS level Further Mathematics)

Depending on the specification followed and modules studied some students who have A level Further Mathematics will also know some: Group Theory, Vector Spaces, Fields, Differential Geometry, Mutivariable Calculus, Differential Equations (including second order), Numerical Methods.

Some students, particularly those who have not studied Further Mathematics, may not have had the opportunity to have studied certain applied modules, e.g. M2, D2 to name but two, during their time in the sixth form. The following pages give information on the approximate content of the first two modules of each applied strand.

Mechanics

Motion Graphs
- ✓ Displacement-time, distance-time, velocity-time
- ✓ Interpreting the graphs
- ✓ Using differentiation and integration

Constant Acceleration and "SUVAT" Equations
- ✓ Introduction to the variables
- ✓ Using the variables
- ✓ Standard properties
- ✓ Use in solving equations

Projectiles
- ✓ Finding the maximum height, range and path of a projectile

Centre of Mass
- ✓ Uniform bodies (symmetry)
- ✓ Composite bodies
- ✓ Centres of mass when suspended

Variable Acceleration
- ✓ Using differentiation in 1 and 2-D
- ✓ Using integration in 1 and 2-D

Uniform Motion in a Circle
- ✓ Angular speed
- ✓ Acceleration
- ✓ Horizontal circle, conical pendulum

Newton's Laws in 1 dimension
- ✓ Motion in a horizontal plane
- ✓ Motion in a vertical plane
- ✓ Pulleys
- ✓ Connected bodies

Vectors and Newton's Laws in 2 Dimensions
- ✓ Resolving forces into components
- ✓ Motion on a slope (excluding and including friction)

Collisions
- ✓ Coefficient of restitution
- ✓ Conservation of linear momentum
- ✓ Impulse
- ✓ Calculations involving a loss of energy

Equilibrium of a Rigid Body
- ✓ Moment about a point
- ✓ Coplanar forces
- ✓ Toppling/sliding

Energy, Work and Power
- ✓ Work done
- ✓ Gravitational Potential Energy
- ✓ Kinetic Energy
- ✓ Conservation of energy
- ✓ Power (force does work)

Statistics

Correlation and Regression
- ✓ Product moment correlation
- ✓ Spearman coefficient rank correlation
- ✓ Independent and dependant variables
- ✓ Least squares regression
- ✓ Scatter diagrams

The Binomial Distribution and Probability
- ✓ Probability based on selecting or arranging
- ✓ Probability based on binomial distribution
- ✓ Expected value of a binomial distribution
- ✓ Expected frequencies from a series of trials

Exploring Data
- ✓ Types of data
- ✓ Stem and Leaf diagrams
- ✓ Measures of central tendency and of spread
- ✓ Linear coding
- ✓ Skewness and outliers

Normal Distribution
- ✓ Properties (including use of tables)
- ✓ Mean and variance
- ✓ Cumulative distribution function
- ✓ Continuous random variables (probability density function and mean/variance)
- ✓ As an approximation to binomial or Poisson distributions
- ✓ t-distribution

Chi-squared
- ✓ Introduction
- ✓ Conditions

Data Presentation
- ✓ Bar charts, pie charts
- ✓ Vertical line graphs, histograms
- ✓ Cumulative frequency

Discrete Random Variables
- ✓ Expectation and variance of discrete random variables
- ✓ Formulae extensions E(aX+b)

Probability
- ✓ Measuring, estimating and expectation
- ✓ Combined probability
- ✓ Two trials
- ✓ Conditional probability
- ✓ Simple applications of laws

Hypothesis Testing
- ✓ Establishing the null and alternate hypothesis
- ✓ Conducting the test and interpreting the results
- ✓ Use of the binomial or normal distribution
- ✓ Type 1 and type 2 errors

Poisson Distribution
- ✓ Properties (including use of tables)
- ✓ Mean and variance
- ✓ Use as an approximation to binomial distribution

Sampling/ Estimation
- ✓ Randomness in choosing
- ✓ Sample means and standard errors
- ✓ Unbiased estimates of population means
- ✓ Use of central limit theorem
- ✓ Confidence intervals

Decision

Graphs
- ✓ Graphs

Networks
- ✓ Prim
- ✓ Kruskal
- ✓ Dijkstra
- ✓ TSP
- ✓ Route inspection
- ✓ Network flows

Critical Path Analysis
- ✓ Activity networks
- ✓ Cascade charts

Game Theory
- ✓ Game theory
- ✓ Using simplex

Optimisation
- ✓ Matchings
- ✓ Hungarian algorithm
- ✓ Transportation problem
- ✓ Dynamic programming

Algorithms
- ✓ Communicating
- ✓ Sorting
- ✓ Packing
- ✓ Efficiency and complexity

Linear Programming
- ✓ LP graphical
- ✓ LP simplex
- ✓ Two stage simplex

Simulation
- ✓ Monte Carlo methods
- ✓ Uniformly distributed random variables
- ✓ Non-uniformly distributed random variables
- ✓ Simulation models

Logic and Boolean Algebra
- ✓ Logical propositions and truth tables
- ✓ Laws of Boolean algebra
- ✓ Combinatorial circuits and switching circuits

5.4 Important dates for Mathematics (Authored by Roger Porkess – up to 2010)

Year	Up to 16	16-19
Earlier	School certificate	Higher school certificate
1950	Start of O level	
1951		Start of A level
1962	First CSE examinations	
1969		Proposal by SCUE to replace A levels with Q and F levels (later rejected)
1970s	Introduction of 16+ examinations combining CSE and O level (running alongside both)	
1973		Proposal by the Schools Council to replace A levels with N and F levels (later rejected)
1973-74	Raising of the school leaving age to 16	
1978	Calculators allowed in O level	
1982	Cockcroft Report	
1983		First A level common cores for mathematics and sciences
1986		New syllabuses based on core
1986-88	O level and CSE replaced by GCSE	
1987		Norm referenced grading of A levels replaced by criterion referenced grading
1988	Coursework in GCSE Mathematics (not compulsory at first)	Higginson Report recommends wider sixth form curriculum (instantly rejected)
1989	Introduction of the National Curriculum	
1990		First teaching of first modular A level syllabus (MEI Structured Mathematics)
1992	First teaching of revised GCSEs to assess the National Curriculum	
1993	Externally set and teacher marked National Curriculum tests at ages 7, 11 and 14	Second A level common cores for mathematics and sciences First core for AS Mathematics
1994	Teacher boycott of marking National Curriculum tests	
1994		First teaching of new syllabuses based on new cores Almost all syllabuses are modular
1996		The Dearing Review on 16-19 Education
1997	New rules require exactly 3 tiers of entry. Several syllabuses (e.g. GAIM and MEI are lost)	New A level syllabuses developed but put on hold following the general election

1999	National Numeracy Strategy (primary schools)	New common cores at AS and A level, now known as subject cores
2000		First teaching of new syllabuses to conform with Curriculum 2000
2000		Work starts on an MEI pilot programme to foster Further Mathematics
2001	Key Stage 3 National Strategy Framework for teaching mathematics to Years 7, 8 and 9 Introduction of data handling coursework at GCSE	Very poor results at AS level Marked drop in retention rate for full A level
2002		Large reduction in numbers taking A level mathematics
2003		New subject cores for AS and A level Mathematics
2004	The Smith Report: Making Mathematics Count	
2004	The Tomlinson Report on 14-19 Curriculum and Qualifications Reform (mostly rejected)	
2004		First teaching of new mathematics syllabuses, now known as specifications, introduced to overcome the problems caused by Curriculum 2000
2005		DfES rolls out the MEI Further Mathematics programme as the Further Mathematics Network (still run by MEI)
2006	First teaching of GCSE syllabuses that have changed from 3-tier to 2-tier	Uptake starts to rise under the new specifications
2007	First teaching of GCSE Mathematics with no coursework	
2009	Key Stage 3 National Curriculum tests discontinued	Uptake is the highest since 1989
2009		The Further Mathematics Network is replaced by the Further Mathematics Support Programme (still run by MEI)
2010	New GCSE syllabuses to start	
2010	Pilot of twin GCSEs to start	

Changes since 2010 (when the original document was written):

At the 2010 General Election, a new government was formed, with a Conservative/Liberal Democrat coalition replacing a Labour government that had been in power for 13 years. This signalled a time of significant change in educational policy. Implementation of such widespread change followed varied timescales, some were immediate, i.e. the 2010 Academies Act, which sought to increase the number of Academy schools, and some took longer, i.e. changes to GCSEs and A levels.

Year	Up to 16	16-19
2011		Uptake of AS and A level Mathematics and Further Mathematics continues to grow
2012		Uptake of AS and A level Mathematics and Further Mathematics continues to grow
2013		Uptake of AS and A level Mathematics and Further Mathematics continues to grow
2014	Teaching of revised National Curriculum for Mathematics begins	Uptake of AS and A level Mathematics and Further Mathematics continues to grow
2014	New network of 'Maths Hubs' (approx 35) created across England	
2015	First teaching of new GCSE Mathematics with grades 9-1 instead of A*-G	First teaching of Core Mathematics qualifications Uptake of AS and A level Mathematics and Further Mathematics continues to grow
2016		First examination of Core Mathematics qualifications
2017	First examination of new GCSE Mathematics	First teaching of new AS and A level Mathematics and Further Mathematics

centre for excellence
sigma
in mathematics & statistics support

MEI Innovators in Mathematics Education

Loughborough University

sigma – Centre for Excellence in Mathematics & Statistics Support
Mathematics Education Centre
Loughborough University Leicestershire LE11 3TU United Kingdom
T +44 (0)1509 227460

MEI – Monckton House Epsom Centre White Horse Business Park
Trowbridge Wiltshire BA14 0XG
T +44 (0)1225 776776