Learning to love maths via e-assessment

– what a title!!!!!

Martin Greenhow

Mathematical Sciences

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Does it count?

– embedding CAA in an effective way

Martin Greenhow

Mathematical Sciences

Brunel University

A dilemma

1. Students do things only for marks (from school?)
2. They work to the next test only (even missing lectures!)
3. So individual lecturers need to set plenty of assessments in order to get their share of students' attention/engagement (else they fail the module)
4. So students have no time for reflection and understanding
5. Go to 1) above!

Result: the course is no fun!

CAA at Brunel University

- Javascript, MathML, SVG, web delivery
- Questions database spans GCSE, A-level, undergraduate topics, ADULT LEARNERS/employability aptitude
- >3500 questions in the database span MC, NI, RNI, PNI, TFU, MR, NI+confidence, Revealed MC, drag&drop etc. (no free-form maths input ... yet): PROS/CONS?
- Hundreds of users take circa 30,000 tests p.a. Students from Maths, Economics, Electrical and Electronic Engineering, Computing, Foundations of Engineering, Foundations of IT, PGCE and Sports Science, even MSc student diagnostics. Much commonality

Adult numeracy?

This week Mavis started her new job as a sales assistant in a huge toy store. The pictogram below shows the number of toys she sold in her first week.

Monday	
Tuesday	
Wednesday	
Thursday	
Friday	
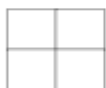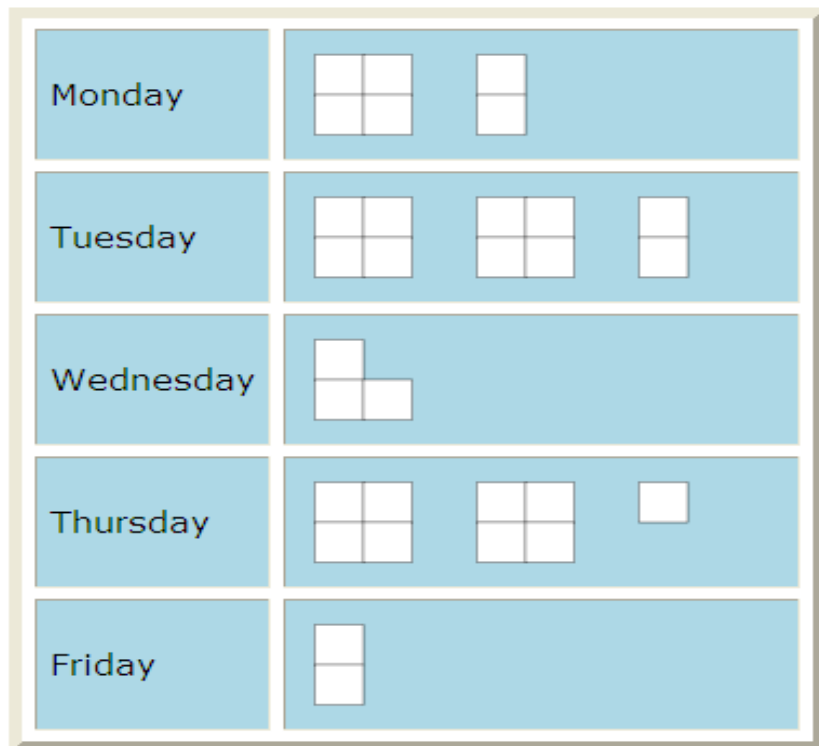
Key:
represents a box of 20 toys.

How many toys did Mavis sell on Thursday?

Submit

start   Shared Documents   Presentation.ppt [Co...   Question Ma

- Pictogram changes
- Scenario changes
- Notice the key -> mal-rule
- 50% of British adults have the numeracy skills required of a 7-year old ... a skills shortage that is holding back the economy

CAA in practice – a way out?

- Low-stakes summative assessment - invigilated or not.

- Preferred scheme: best-ever mark from their first 5 attempts counts towards their module mark; not invigilated and group work is allowed/encouraged. **Exam pass required!**

- **Blended assessment (CAA, traditional exams, labs, reports & essays …) Why drive a car?**

- **Mostly we aim to promote learning … we don't really need more marks!**

MATHS TESTS 1-5

Prices Currently

. Other Tests - £5

. Matrices Part 1 - £10

. Matrices Part 2 - £10

. **All 5 tests = £30 (save £5)**

Flexible Availability - Living on Campus

PM ME IF Interested

SC Milen_Yadav

90% and Below = Full Refund

Milen Yadav

2 slots available, prices will rise closer to the deadline   13:47

PM ME If Interested, all you need to provide is your computer login and a picture of the results will be sent through both Whatsapp and Email.

13:53

Mathematics for Economics
(year-by-year changes in assessment)

2006-07, 2007-08:   no CAA; A and non-A cohorts – results indistinguishable; two class tests, one 3-hour exam set by economics staff <u>(10 days - times 2?)</u>

2008-09:   CAA introduced; 3-hour exam only
                    (2 days - times 2?)

2009-10, 2010-18:   one cohort, admission AS level mathematics; exam pass mandatory; 2-hour exam only (set by MG to 2015)
                    (3 days!!!!!!)

Maths for Economics year-on-year results (% in grade/% mark) circa 400,000 questions done
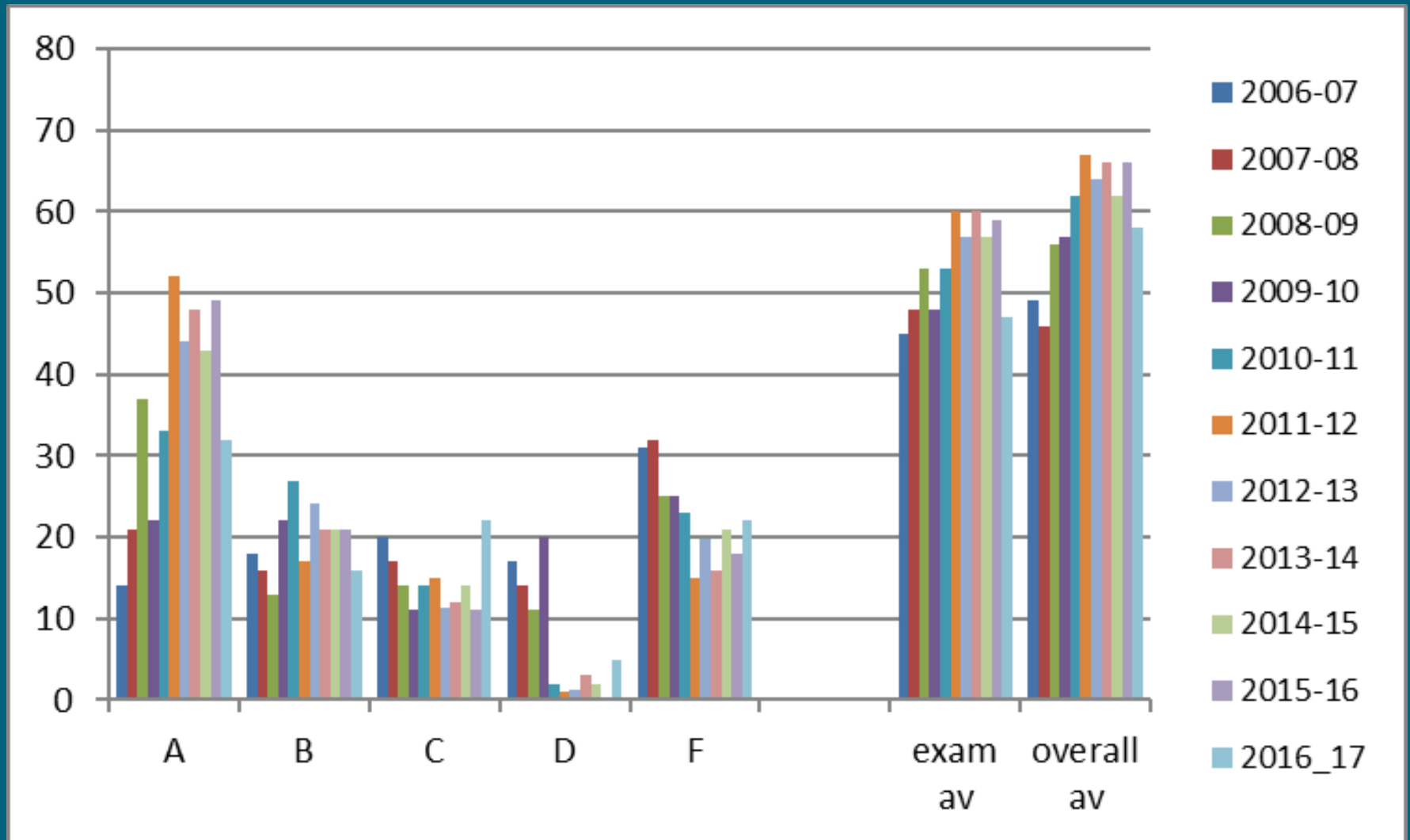
Effective learning + Economies?

-

Simplifying the logistics of setting tests, particularly in-course or continuous assessment tests of large cohorts (up to several hundred in an HE context). Depending on the embedding scenario chosen (see below), CAA can remove many of the administration tasks such as timetabling, booking rooms, informing students, making arrangements for students with special needs, repeat tests for those unable to do the first test instance . . . and finally the whole process again for resit tests.

Effective teaching /economies!

Endless source of reverse-engineered questions useful for lecture notes, problem sheets (if used) and first part of exam questions, followed perhaps by objective open questions to do with:
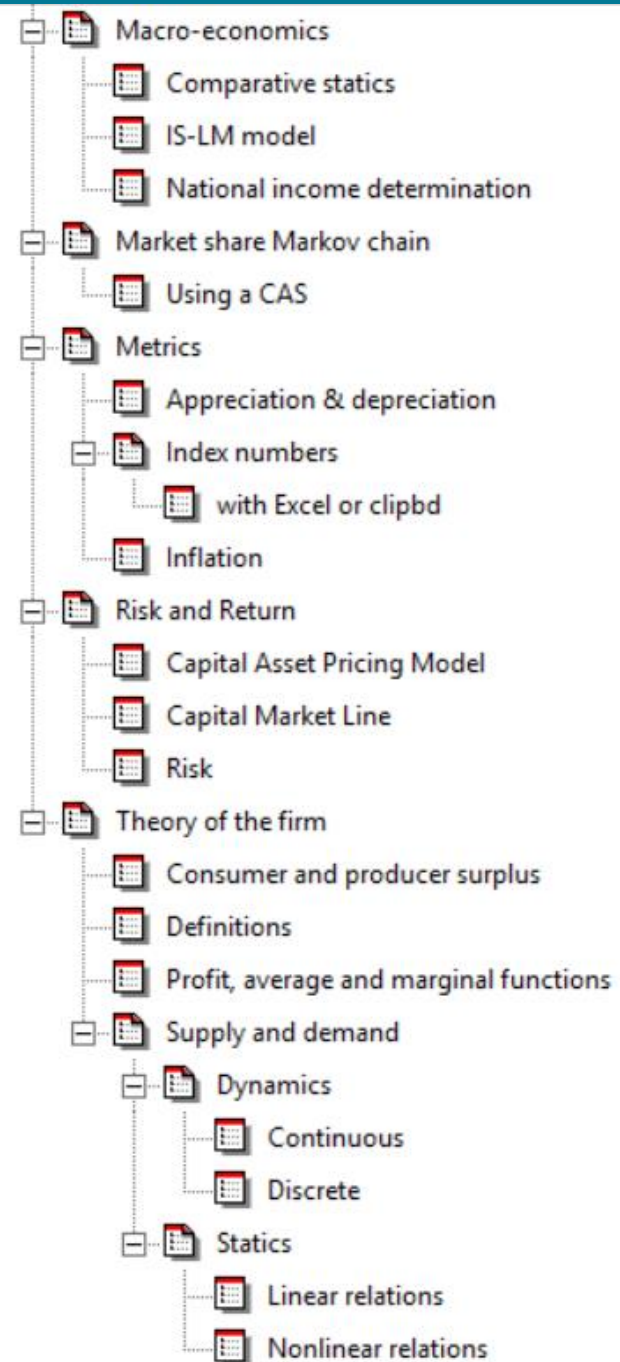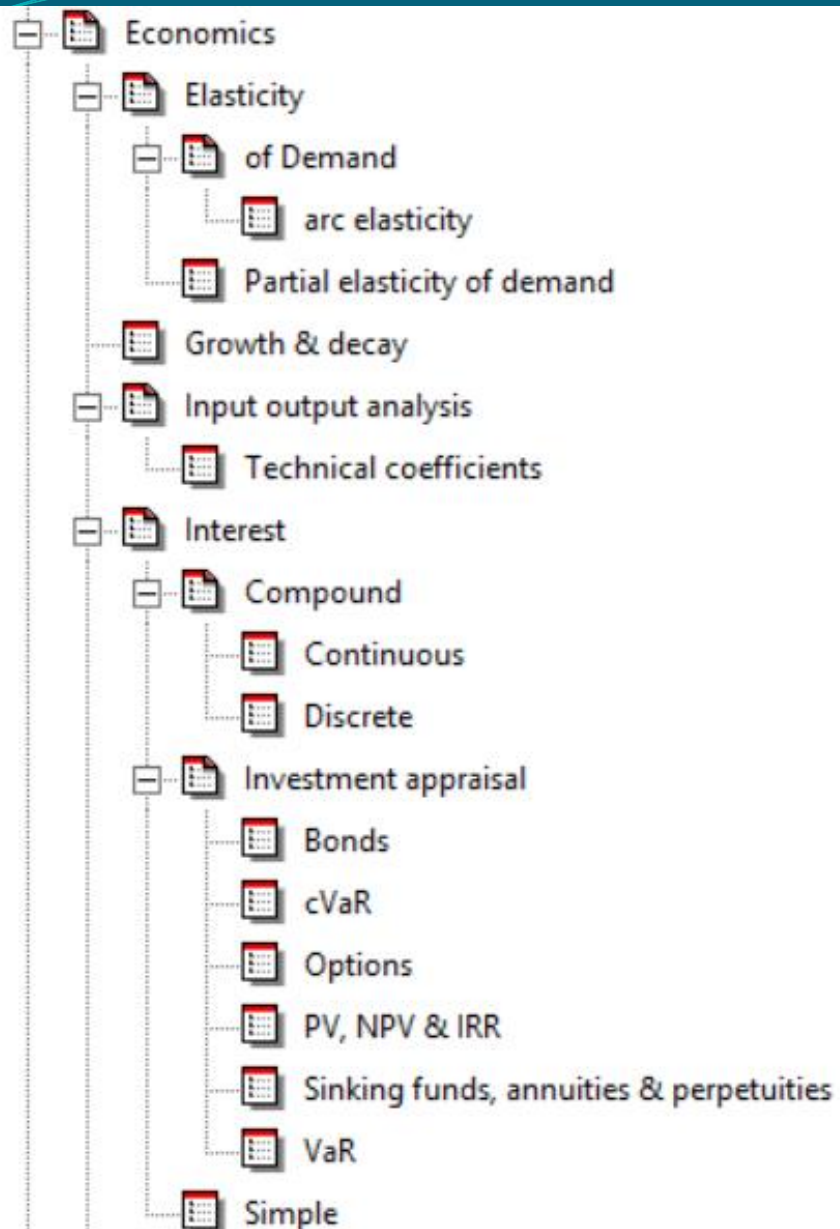
Modelling,

Proof

Interpretation

Critiquing

Extension ...

Metal HEA Project

- Used design methodology and experiences to set objective maths for economics

- Questions – not tests; staff can pick & mix

- Backed up by videos, case studies & simulations; questions sometimes link to these

- Has since been extended considerably – often by Final Year Project students doing Financial Maths

- Classified by topic and tagged for Search Engine

- Main feature: FULL FEEDBACK which students study (a lot)

Topics in Economics

Economics
- Elasticity
  - of Demand
    - arc elasticity
  - Partial elasticity of demand
- Growth & decay
- Input output analysis
  - Technical coefficients
- Interest
  - Compound
    - Continuous
    - Discrete
  - Investment appraisal
    - Bonds
    - cVaR
    - Options
    - PV, NPV & IRR
    - Sinking funds, annuities & perpetuities
    - VaR
  - Simple

- Macro-economics
  - Comparative statics
  - IS-LM model
  - National income determination
- Market share Markov chain
  - Using a CAS
- Metrics
  - Appreciation & depreciation
  - Index numbers
    - with Excel or clipbd
  - Inflation
- Risk and Return
  - Capital Asset Pricing Model
  - Capital Market Line
  - Risk
- Theory of the firm
  - Consumer and producer surplus
  - Definitions
  - Profit, average and marginal functions
  - Supply and demand
    - Dynamics
      - Continuous
      - Discrete
    - Statics
      - Linear relations
      - Nonlinear relations

Complete the square of the following expression:

$$7s^2 + 4s + 3$$

~~~~~~Your result~~~~~~

1). Your answer $7\left(s + \frac{4}{7}\right)^2 + \frac{5}{7}$ should have been $7\left(s + \frac{2}{7}\right)^2 + \frac{17}{7}$

GENERAL THEORY	THIS EXAMPLE
$as^2 + bs + c = a\left\{s^2 + \frac{b}{a}s + \frac{c}{a}\right\}$	$7x^2 + 4s + 3 = 7\left\{s^2 + \frac{4}{7}s + \frac{3}{7}\right\}$
$= a\left\{\left(s + \frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{4a^2}\right\}$	$= 7\left\{\left(s + \frac{4}{14}\right)^2 + \frac{3}{7} - \frac{4^2}{4 \times 7^2}\right\}$
$= a\left(s + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$	$= 7\left(s + \frac{2}{7}\right)^2 + \frac{17}{7}$

Study the above solution carefully

Related material

0 out of 1
Your answer is incorrect.
You have the right idea, but in the bracket you need to divide the coefficient of *x* (i.e. *b*) by 2*a* and not just *a*.

Typical algebra MC question

FEEDBACK –> LEARNING RESOURCE

Related material links to stuff on web (Mathcentre, Exam Solutions, PurpleMaths, etc.) but this needs to be SECTIONED UP and be there (not U-tube)

What do we need to teach in class? Probably this, probably not gen. quad. formula.

MCQs a bad image?

- MCQs merely ask students to spot the correct answer when they see it.
- They are very heavily 'scaffolded' with the key and distracters suggesting the form or size of the answer, e.g., not taking the square root of the variance would give a distracter that is unrealistically large for the standard deviation.
- Partial knowledge may eliminate some distracters and encourage guessing, for example by using symmetry (a good skill, but probably not the one intended to be tested by the question).
- Trial values such as x=0, 1, 2 . . . might be used until all but the correct answer is eliminated.
- The question might be done backwards, e.g., differentiating all the options rather than integrating the integrand in the question.
- Pattern spotting might be employed where the student looks for commonality in parts of the options. For example, what are the coordinates of a calculated point in space, with options (1,1,1), (1,1,0), (1,0,1) and (0,1,1) suggest that (1,1,1) is the correct answer.

But they BUILD CONFIDENCE!

- Example: Differentiate $\cos(2x)$
- Mal-rules? Pick up usual errors easily …

Differentiate $\cos(x^2)$ … needs a human to explain? Well maybe not!

Question Manager

File Edit View Topic Question Tools Help

Confidence appraisal\Linear algebra

No Knowledge : No Confidence : Little Confidence : Some Confidence : Confident : Very Confident :

Topic	Your response
Given $C = \begin{bmatrix} 5 & 4 \\ 9 & 4 \\ -7 & 1 \\ 3 & -5 \end{bmatrix}$, find 5**A**.	
Given $A = \begin{bmatrix} 7 & 4 \\ 1 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} -8 & -9 \\ -6 & 4 \end{bmatrix}$, find $A - B^T$.	
Given $A = \begin{bmatrix} 7 & 4 \\ 1 & -5 \end{bmatrix}$ and $C = \begin{bmatrix} 5 & 4 \\ 9 & 4 \\ -7 & 1 \\ 3 & -5 \end{bmatrix}$, find, if possible, **AC** and **CA**.	
Write $\begin{bmatrix} -5 & 2 & 3 \\ -1 & 1 & 2 \\ 2 & -3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$ as three simultaneous equations.	
Write $\begin{bmatrix} 8 & 3 & 8 \\ -4 & -7 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \end{bmatrix}$ as two simultaneous equations in x and y and solve for each in terms of z.	
Given $\begin{bmatrix} -5 & 2 & 3 \\ -1 & 1 & 2 \\ 2 & -3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$, find z using Cramer's rule.	
Given **C = AB** where **A** is a given 5×7 matrix and **B** is a given 7×2 matrix, determine, if possible, $c_{3,2}$ and $c_{2,3}$.	
Given $Q = \begin{bmatrix} -5 & 2 & 3 \\ -1 & 1 & 2 \\ 2 & -3 & 2 \end{bmatrix}$, find tr(-3Q).	

Tree panel (left)

- Confidence appraisal
  - Algebra
  - Boolean algebra
  - Calculus
  - Functions
  - Linear algebra
  - Mechanics
  - Numbers
  - Statistics & probability
- Coordinate geometry
- Decision mathematics
  - Linear programming
    - Feasible regions
    - Optimisation (any method)
    - Simplex method
- Differentiation
  - Algebraic functions
  - Chain rule
  - Elementary functions
  - Implicit
  - Logarithmic
  - Optimisation - single variable
  - Parametric
  - Partial differentiation
  - Polynomials
  - Product rule
  - Quotient rule
  - Rational functions
  - Related rates
  - Series expansions

Description list

Description		Undergra...	A-level	GCSE	Difficulty	templa
Vectors; XC6		Level 1	FP3		Hard	
Vectors; XC6		Level 1	FP3		Interme...	
Matrices; XC6		Level 1	FP3		Easy	defaul
Matrices; XC6		Level 1	FP3		Interme...	defaul
Determinants; XC6		Level 1	FP3		Interme...	defaul

Press F1 for help

Print this screen Colours & Fonts Mathletics

1 of 1

Bruce is trying to decompose the following rational function into the sum of partial fractions, but he may have made a mistake.

If there is a mistake in Bruce's workings shown below, in which line was the mistake first made?

line 1: $\dfrac{10x^3 - 26x^2 - 8x - 8}{-8x^4 + 2x^3 - 23x^2 + 12x - 15} = \dfrac{10x^3 - 26x^2 - 8x - 8}{(-2x^2 - x - 5)(4x^2 - 3x + 3)}$

line 2: $\dfrac{10x^3 - 26x^2 - 8x - 8}{-8x^4 + 2x^3 - 23x^2 + 12x - 15} = \dfrac{Ax + B}{(-2x^2 - x - 5)} + \dfrac{Fx + G}{(4x^2 - 3x + 3)}$

line 3: $10x^3 - 26x^2 - 8x - 8 = (Ax + B)(4x^2 - 3x + 3) + (Fx + G)(-2x^2 - x - 5)$

line 4: $10 = 4A - 2F$

line 5: $-26 = -3A + 4B - F - 2G$

line 6: $-8 = 3A - 3B - 5F - G$

line 7: $-8 = 3B - 5G$

line 8: $\therefore A = 5, B = -1, F = 5$ and $G = 1$

line 9: $\dfrac{10x^3 - 26x^2 - 8x - 8}{-8x^4 + 2x^3 - 23x^2 + 12x - 15} = \dfrac{5x + 1}{(-2x^2 - x - 5)} + \dfrac{5x - 1}{(4x^2 - 3x + 3)}$

Please *input the line number where the mistake first occurs, or input 0 if there is no mistake.*

The error is in line ...

Reverse engineering
Scenarios
MathJax

A positive quantity Q is known to depend on two positive variables as follows:

Mathematically we can write this as: $Q = k \, \dfrac{R}{S^3}$

The following claims are made.
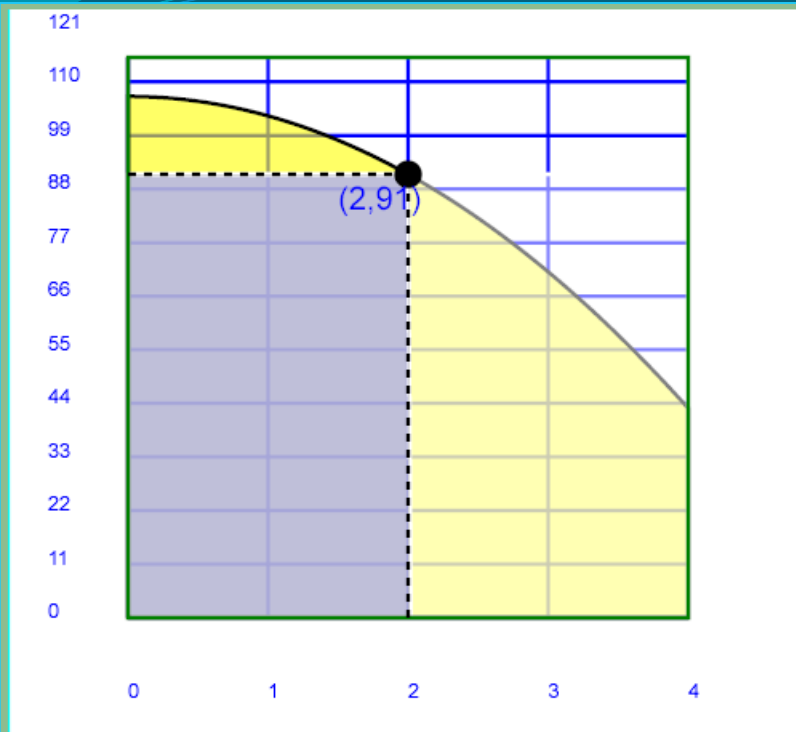
*If you think the claim is **true, input T**.*
*If you think the claim is **false, input F**.*
*If you think the claim is **undecidable** on the basis of the information given, i*

Claim	T, F or U ?
If R decreases and S increases, Q decreases.	
If R decreases and S decreases, Q stays the same.	
If R decreases and S stays the same, Q increases.	
If R increases and S decreases, Q decreases.	

- Tests understanding (I think …)
- This question context (not fully shown) makes it clear that k > 0
- What if k < 0 ?

Feedback - contextualised

The consumer surplus is given by the yellow area in the above graph; this is the definite integral under the curve from 0 to 2 minus the area of the blue rectangle of height 91 and width 2 i.e.

$$CS = \int_0^2 \left(107 - 4Q^2\right) \, dQ \quad - \quad 91 \times 2$$

$$= \left[107Q - \frac{4Q^3}{3}\right]_0^2 \quad - \quad 182 = 21.333$$

Related material

- economics question
- part of the formative feedback
- SVG diagram realised according to the random parameters in the question
- related material button links to any web resource (via centrally-held lookup array)

Demand for oil is currently 59000 billion units per year, but is set to fall by 13% annually. If current world stocks stand at 393000 billion units, after how many years will this resource completely exhausted?

~~~~~~Your result~~~~~~
Your answer 333 years, should have been 15.

At the end of the year 1 we will have used 59000 units.

At the end of the year 2 we will have used another 59000(1-0.13) units.

At the end of the year 3 we will have used another $59000(1-0.13)^2$ units.... and so on until

At the end of the year n we will have used another $59000(1-0.13)^{n-1}$ units.

Adding these up gives the series:

$$59000 + 59000(1-0.13)^1 + ... + 59000(1-0.13)^{n-1}$$

i.e. a geometric progression with first term 59000

common ratio $(1-0.13)$

and the number of terms is *n* which you needed to find.

Using the general formula for the sum of a G.P. $S_n = \dfrac{a(r^n - 1)}{r - 1}$

Setting this equal to the world reserves gives $\dfrac{59000(0.87^n - 1)}{0.87 - 1} = 393000$

To solve this for *n* we need to simplify: $(0.87^n - 1) = \dfrac{393000(0.87 - 1)}{59000} \approx -0.865932$
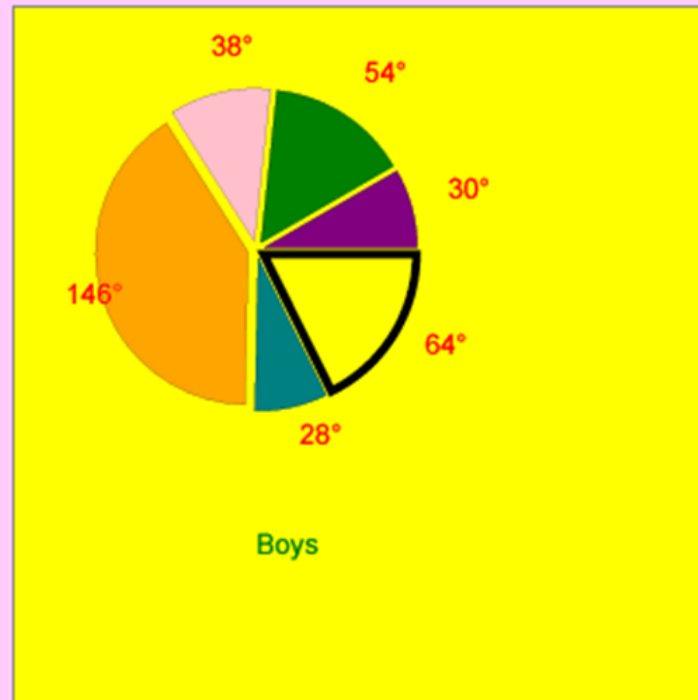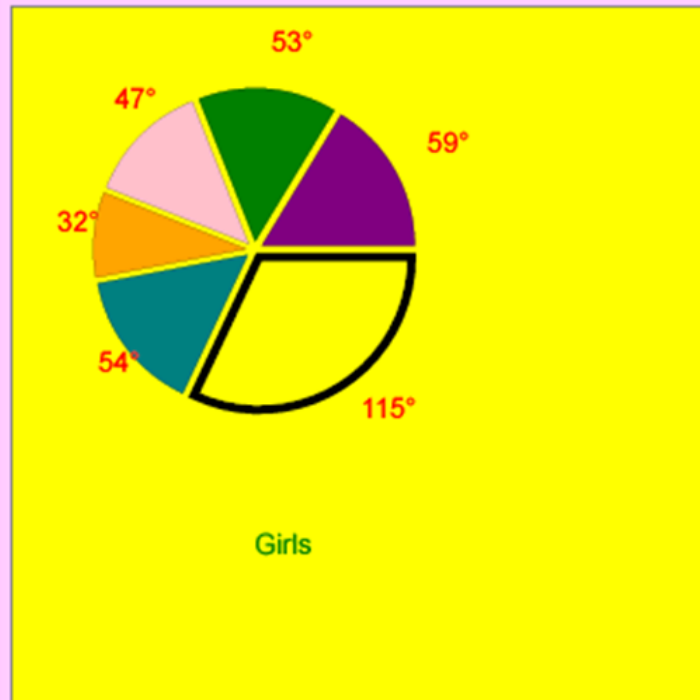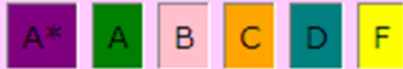
Adding 1 to both sides and taking logs gives: $n = \dfrac{\log(0.134068)}{\log(0.87)} = 14.428970$

so we have completely exhausted the world reserves of oil after 15 years.

DeSTRESS JISC Project

- Builds on the Metal question design methodology and experiences to test basic stats for social sciences

- Released new material end August 2011

- Hand calculation with realistically-sized data sets – solution to link with external software such as Excel, SPSS

- Real data? Data cannot be 'designed' to have certain features or not. Problems with keeping the data and answers current without accessing live web sites (problems of access and communication with the marking scheme).

- Another challenging area is the interpretation of charts and graphs

George works for an exam board and is looking at the GCSE maths results. He wants to examine any gender bias by taking 300 candidates of each gender. The results are presented in the pie charts below. (Due to rounding, at times the values around the pie chart may not add to 360°)

| A* | A | B | C | D | F |

Girls

Boys

Calculate the difference in number between girls and boys who achieved grade F.

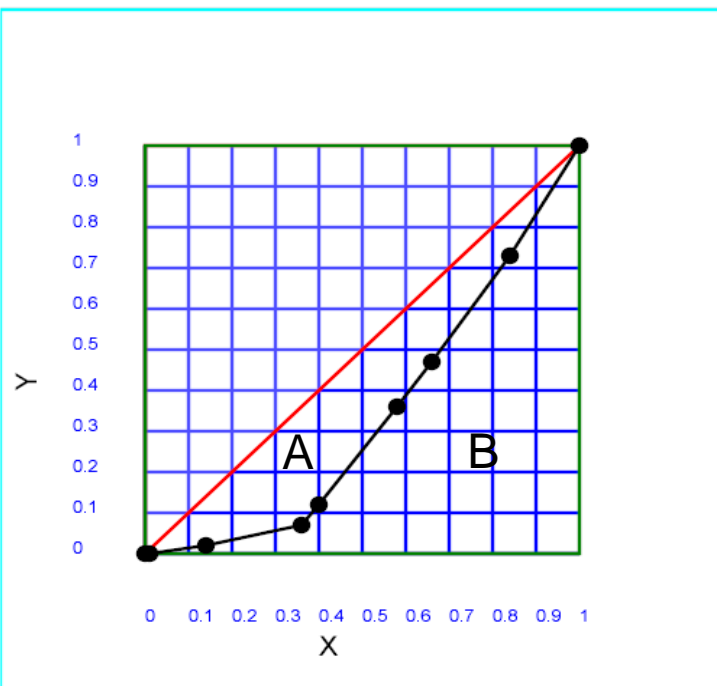Round your **final** answer to the nearest whole number.

Descriptive stats … lots of randoms here including stated grade and dynamic diagrams via SVG

The table below gives income data i.e. coordinates on the Lorentz curve for a certain country. Calculate the Gini coefficient assuming linear

Cumulative percent of households	1	14	36	40	58	66	84	100
Cumulative share of national income	0	2	7	12	36	47	73	100

Economics applications\Percentages\Index numbers

The Gini coefficient is the ratio of the area A above the black Lorentz curve (actually a ploygon here) and below the red line of equality (which has area = 1/2 i.e. the area of the triangle of width and height 1). This is then simply 2A. The curves are shown below. The line of equality is when everyone gets the same income and then G = 0. At the other extreme, suppose just one household gets all the national income (of a large population) then the Lorentz curve would be alone the x-axis until x=1 when it would shoot up to y=1. That curve would have no area underneath it so A would equal 1/2 and G would then be 1.

Small Gini coefficients therefore imply a more equal society, although the income Gini coefficient has nothing to do with the wealth distribution - for that we would need different data and then calculate the wealth Gini coefficient. Typically under-developed countries have larger income and wealth Gini coefficients where the money is in the hands of an elite whilst the majority of the population lives in relative poverty. However, note the word *relative* here; the Gini coefficient tells us nothing about absolute standards of income, merely how equal incomes are (or not). In the UK the income Gini coefficient has increased rapidly to the present date (August 2011) and one could ask if this is at least partly related to recent rioting!

Gini coefficient

$G = A/(A+B) = 2A$

A statistics question - tables or formula?

Statistics\Distributions\Binomial\Cumulative probability\Decontextual\by formula or 4dp tables

From the Cumulative Binomial distribution table below find and click the probability stated below, where X follows a binomial distribution and has the given value of n. The table below provides corresponding probabilities for *X* less than or equal to stated values of *x*.

You can choose your answer from the Binomial distribution table by clicking your mouse. You will not be able to change your mind after you click on an answer, so make sure you choose your answers carefully before clicking. This question holds 2 marks, where full marks can be achieved by selecting the correct answer at the first attempt.

n= 20

Table

Message from webpage

Your selected answer is incorrect, you have one last chance to try this question.

OK

$P(X \leq 10 | p = 0.25)$ has the value of (click on the table)

P=	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n=5,x=0	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681	0.116	0.0778	0.0503	0.0312
1	0.9774	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.337	0.2562	0.1875
2	0.9988	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931	0.5
3	1	0.9995	0.9978	0.9933	0.9844	0.9692	0.946	0.913	0.8688	0.8125
4	1	1	0.9999	0.9997	0.999	0.9976	0.9947	0.9898	0.9815	0.9688
n=10,x=0	0.5987	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.006	0.0025	0.0025
1	0.9139	0.7361	0.5443	0.3758	0.244	0.1493	0.086	0.0464	0.0233	0.0233
2	0.9885	0.9298	0.8202	0.6778	0.5256	0.3828	0.2616	0.1673	0.0996	0.0996
3	0.999	0.9872	0.95	0.8791	0.7759	0.6496	0.5138	0.3823	0.266	0.266

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JISC

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Search   Go

Favorites   Suggested Sites ▾   Free Hotmail   Web Slice Gallery ▾

Page ▾   Safety ▾   Tools ▾

maths e.g.

maths e.g.

- Algebra
  - Adding polynomials
  - Algebraic fractions
  - Complete the square
  - Dimensional analysis
  - Expanding brackets
  - Factorisation
  - Indices
  - Linear equations
  - Modelling
  - Pascals Triangle
  - Proportionality
  - Quadratic equations
  - Rearranging equations
  - Roots & factors of polynomials
  - Simultaneous equations
  - Summations
  - Terminology
  - Understanding expressions
  - Algebric functions
- Differentiation
- Integration
- Matrices
- Numbers

Questions in topic and   Related material

1	ma+nb=0; m,n +ve MC
2	inferences for Q=-R^n+S^m given equation; 4TFUSP
3	inferences for Q=-R^n-S^m given equation; 4TFUSP
4	inferences for Q=k*R^n/S^m; 4TFUSP
5	inferences for Q=k*R^n/S^m given equation; 4TFUSP
6	inferences for Q=R^n+S^m given equation; 4TFUSP
7	inferences for Q=R^n-S^m given equation; 4TFUSP
8	ma*nb=0; m,n +ve MC

Teacher interface – shop on Amazon?

➢ Javascript, MathML and SVG provide a rich environment for setting objective questions

➢ Positive effects on students' perceptions and on exam performances

➢ Widely applicable database of questions

➢ Good source of reverse-engineered questions for all, especially teachers

➢ MathsE.G. Try it at:

http://www.mathcentre.ac.uk:8081/mathseg/

http://www.mathcentre.ac.uk:8081/mathsegteacher/

works on all browsers, PC & Mac, I-pads and smart phones, no link to VLEs yet!