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Editor's Notes

The fifth CETL-MSOR conference was held 6-7 September 2010 at Birmingham University. There were – as usual – a large number of excellent presentations made which will appear (or may have already appeared) 'in print' in a variety of formats. These Proceedings are but a selection, which – to parody the Clio car advertisement – is 'small but perfectly informed'.

Although I was unable to attend myself (I was getting high in South America), those who attended have told me it was a most enjoyable and worthwhile event like its predecessors. I am indeed sorry to have missed the many interesting talks and workshops.

For these Proceedings a two-stage refereeing system was used, with a small team sifting through the potential papers and then a larger team providing expert feedback on those selected. My thanks goes to all the referees for making my task that much more straightforward.

Conference Papers in these Proceedings

Of these eleven papers, most relate to foundation or Level 1 HE mathematics, and many are concerned with various approaches to, and uses of, assessment.

Ahmed and Love report on their experiences at a university which has a flexible entry system whereby most Level 1 students study three subjects which may span the faculties. This leads to a major challenge in providing mathematics support when specialists and non-specialists are enrolled on the same module. The paper outlines two modes of support provided and discusses their efficacy, based on an analysis of academic success and student feedback.

Barton and Rowlett report on their use of an audience response system for whole class quizzes in a Level 1 mathematics module aimed at encouraging the students to keep up to date with their lecture notes. The paper reports on findings from a student feedback questionnaire seeking their views on, and reaction to, the technology. Based on this investigation the authors make recommendations on how to best use such devices.

Bradshaw, McCartney and Mann report on a collaborative project which produced both audio-based and text-based resources on the history of mathematics, designed to support the teaching (and learning) of mathematics. This included the innovative delivery by MP3 player. The paper reports on the benefits and issues identified, which they obtained through observation and students' reactions.

Crompton reports on his investigation into whether Level 1 students using a computer algebra system adversely affects their ability to solve mathematical problems (by weakening their basic skills) or indeed enhances their ability. The paper describes the scenario where students use the package to assess their work ('self-grading') and so obtain feedback. He provides a statistical analysis to support his findings.

Grehan, O'Shea and Mac an Baird report on a study enquiring into the reasons why students do or do not engage with mathematics support. This is an important and puzzling issue. They studied Level 1 students who had struggled but passed their mathematics examinations and those who had failed, and found both similarities and some significant differences. The paper concludes with a review of 'coping mechanisms' which students employ.

Lowe and Hasson report on the introduction of on-line interactive computer marked assessment for a distance learning foundation level mathematics module. The style of assessment, its development and its integration within the module are all described, together with student feedback and issues encountered.

McCabe, Williams and Pevy reflect on their experiences following a move from using in-house electronic resources for mathematics to those provided by an educational publisher. The paper reports on a survey on the way Level 1 and Level 2 students taking calculus modules reacted to the new system. This paper raises a number of important questions and goes some way towards providing answers.

Ní Fhloinn reports on a novel project whereby university students worked with local school pupils in a disadvantaged area, aiming to increase the pupils' confidence levels and mathematical standards, and to raise the profile of mathematics within the whole school. A large number of university students from a range of backgrounds and courses volunteered and were involved, and feedback was positive across the board. This paper discusses the practicalities of the scheme and the challenges faced, and assesses benefits which can accrue.

Sangwin reports on a mini-project funded by the HE National STEM programme utlilising the STACK CAA system, whose aim was to take existing diagnostic tests in core mathematics and develop equivalent automatic tests. Such tests play an important part in identifying students at risk, targeting remedial help and designing realistic modules. To those ends, 103 core skills were identified across 17 groups of skills and a battery of questions devised and trialled with Level 1 students. An analysis of findings is presented.

Trott reports on the nature and prevalence of dyscalculia in Higher Education and highlights a software screening tool specifically designed to diagnose this complex syndrome. The paper first defines the disorder, presenting some theoretical perspectives, and then reports on the development, trialling, calibration and implementation of this unique tool.

Zaczek and Greenhow explore issues in setting CAA objective questions for a discrete mathematics foundation module. They demonstrate how random parameters can be used to make all aspects of questions dynamic: scenarios, wording, equations, tables, graphs and diagrams. They also present and discuss finding from the analysis of students' CAA-based examinations.

David Green

Provision and Evaluation of Mathematics Support at the University of Glasgow

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1. Abstract

The flexible faculty-entry system at the University of Glasgow results in the majority of first year students studying three subjects which will often span more than one faculty. Some students will have enrolled in Mathematics as their intending Honours subject; some will be pursuing it as a prerequisite for another degree course such as Physics or Computing Science while others still may have chosen Mathematics to complete their curriculum, having no intention of continuing it beyond first year. This provides an interesting situation where no distinction is apparent in the provision of Mathematics support aimed at a specialist or non-specialist student.

Because of the wide variety of academic backgrounds that students bring to university, combined with their differing degree aspirations due to the faculty-entry system, the level of support that a student may require or benefit from varies greatly. At Glasgow, two types of Mathematics support are available to students studying Level 1 Mathematics: one is through the Maths Adviser based in the Student Learning Service and the other from the Study Support Co-ordinator based within the faculty. There is a degree of overlap in both areas of support. Both services are well-used by the students and receive positive feedback.

2. Provision

Two Mathematics courses are on offer to Level 1 students. The larger of the two classes, with around 500 students, is Module A, and has a prerequisite of Scottish Qualifications Authority (SQA) Higher Maths (or equivalent). This leads onto Module B – if Maths is to continue into Level 2 – or Module C in the second semester.

Module D, with a class of around 120 students, is offered to students with SQA Advanced Higher/GCE A-Level Maths backgrounds. This leads to Module E in the second semester and then to level 2 Maths if the student wishes to pursue the subject further.

A compulsory aspect of the mathematics course is the Skills Test. This test is designed to identify gaps in students' knowledge or highlight which skills they may need to practise to make the most of their undergraduate Maths course. Although the test is zero-credit, it has to be passed satisfactorily to be allowed entry into level 2.

2.1 Maths Support in the Student Learning Service (SLS)

The Maths Adviser provides support to all Level 1 students and other undergraduates studying Level 1 mathematics. One-to-one appointments and drop-in sessions are offered to students along with resources which are available for downloading or printing from Moodle (a free open-source VLE). Some in-course sessions are also provided as part of the first year curriculum in various departments (Biology, Mathematics, Nursing, etc.).

2.2 Study Support Co-ordinator

The Study Support Co-ordinator works closely with academic staff in the Faculty of Information and Mathematical Sciences with the ultimate aim of improving student retention. Throughout the academic year Level 1 Mathematics students are supported in various ways. They receive pre-arrival packs prior to entry and in week zero they are inducted into the faculty and invited to become a member of MacSoc, the student society for mathematics and statistics students. Their academic progress throughout Semester 1 is monitored closely, with any students who appear to be at risk being invited to an informal, confidential chat. Voluntary Peer Assisted Learning (PAL) sessions are available for all students to attend, but especially recommended for those who are struggling.

3. Evaluation

3.1 Pass rates

Pass rates have been increasing over the last 6 years but a marked increase was noticed over the last 2 years. However, it has been felt that the range of factors influencing exam results, both positively and negatively, are too great to draw meaningful conclusions.

Possible reasons for increased pass rates could be:

- Better students, i.e., more students coming to university with higher grades;
- Better prepared students because of pre-arrival materials and extended inductions;
- Smaller tutorial groups compared to those in previous years;
- Maths support reaching out to a wider population through the SLS and faculty PAL sessions.
- Skills Test succeeding in identifying gaps in knowledge and basic skills;
- Academic skills advice from Effective Learning Advisers in the SLS.

Over the last two academic years, the number of students using the SLS maths support service has doubled, as has attendance to PAL sessions.

3.2 Student feedback

Students find both services useful and much positive feedback has been received.

They appreciate being able to speak to the Maths Adviser on a one-to-one basis and have commented on the fact that they "feel comfortable enough to ask basic questions without feeling stupid". Students have also remarked on "feeling more confident" following one or more appointments and like having a quiet place to study within a friendly environment.

With Peer Assisted Learning sessions, students have "enjoyed working informally with other members of the class" and have liked interacting with the facilitator who did the same course as them only a few years ago.

3.3 Interesting findings

- It was found that students who had difficulties in Semester 1 did not necessarily find Semester 2 difficult. This was most probably because students had settled into university life and were getting used to lectures, tutorials and independent study.

- Unfortunately, students were often found to be remarkably unconcerned about not understanding chunks of topics as they could "pick up marks elsewhere in the exam". They seemed to have little understanding of the long-term effects of this in a subject such as Mathematics.
- Students were found to be very resistant to working in groups within the department early in the university year. However, during the exam revision period when the Maths Support Service was extremely busy, study groups were formed quite spontaneously and worked very well.
- There appears to be no competition between the two forms of maths support; a lot of students will use both.

4. Recent developments

Funding has become available to allow a specialist Statistics Adviser to be recruited in the Student Learning Service to provide Statistics support to all level 1 students and other undergraduates studying level 1 Statistics.

The University of Glasgow has recently undergone a major restructuring with the department and faculty structure being replaced with a school and college structure. The faculty Study Support Co-ordinator's role has necessarily changed, and is currently involved in sharing good practice within, and establishing a consistent support framework throughout, the new College of Science and Engineering.

Finally, a large research project is currently investigating the extent of, and the consequences of poorly considered second and third subject choices by first year students. Preliminary findings are confirming the suspicion that the root of some of the retention problems within mathematics originate before the actual teaching begins. Future work will be focussing on the realisation that the applicants need greater support in choosing their Level 1 curriculum, not only their intending degree, and continuing to improve upon the preparation for arrival that is provided for the new students.

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Using an audience response system – what do the audience DO with the feedback?

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Abstract

An audience response system was used for whole class quizzes in a Level One applied mathematics module taken by approximately 180 students, with the aim of encouraging students to keep up to date with lecture notes between lectures. A questionnaire on student behaviour in relation to use of the technology was administered to evaluate the use of technology against this aim. Students who were more engaged with the quizzes took remedial action to a greater extent, even when they answered correctly. Students who were less engaged with the quizzes tended to take remedial action less often and when they did they most often took action by speaking with their friends about the question. Some recommendations for designing successful approaches are given with some questions for further investigation.

1. Background

In a Level One applied mathematics module taken by approximately 180 students, the lecturer was concerned about the extent to which students kept up to date with lecture content during the term, rather than leaving everything until the end in the hope that it would become clearer. To encourage review of notes between lectures, regular whole class quizzes were introduced into fortnightly examples classes.

The University provides a Keepad TurningPoint, http://www.turningtechnologies.com/, audience response system[1] which was used for these quizzes. This consists of a number of handheld devices with digits 0-9 which communicate via radio to a USB dongle connected to a PC, on which is installed software that collects the responses.

An alternative approach would be to conduct a vote through a 'show of hands'. While giving each student the chance to participate in the session, the practical difficulties of counting hands mean this approach does not scale very efficiently. Also, Wit [1] reports students experiencing inhibitors such as "fear of ridicule by the lecturer or their peers" (p. 16) so the lack of anonymity may preclude a comprehensive and honest response. Figure 1 shows a question used in one of the quizzes and Table 1 gives the recorded responses. Using the show of hands method in this example, with 84% of the audience choosing option C, it seems likely that the 5% who were waiting for options D and E would put their hand up with option C, masking any lack of knowledge or misconception. It is also possible the 1% choosing option A and some of the 9% choosing B would not have put their hand up with such small minorities. Anonymity can be addressed somewhat through holding up more discreet coloured cards matching different responses, though not completely and the problem of counting remains. A completely anonymous solution exists in Optical Mark Reader (OMR) sheets. These are grids of dots and the students mark out their responses on appropriate dots. These sheets are then fed through a machine to produce a combined set of responses. The length of time taken by this process means the OMR sheets cannot be used to give instantaneous feedback in class and the lecturer's previous experience of this technology indicated students did not always return their completed OMR sheets at the end of a session.

Question: What is the path given by **x** = (e^tsin2t, e^tcos2t, 0)?

A: Line
B: Circle
C: Spiral
D: Parabola
E: Don't know

Figure 1: an example question from one of the whole class quizzes.

Option	Percentage of responses
A	1%
B	9%
C	84%
D	4%
E	1%

Table 1: answers recorded by audience response system to question given in Figure 1.

An audience response system is thought to bring two-way communication in lectures and provide an active learning opportunity to every member of the audience [2]. Wit [1] says that, having chosen some answer, "the psychological investment in that answer turns the student from a passive attendee into an active participant for whom the outcome has some emotional value" (p. 16). This is a form of engagement and personal interaction, even in an audience of many, which can be missing from more automated assessment/feedback systems. The two-way communication means the lecturer gets feedback on misunderstandings and gaps in the students' knowledge. In this instance the lecturer planned to use the feedback provided by the system to incorporate areas, in which a sizeable number of students were not answering correctly, into future lectures.

One approach to using audience response systems shown to have some success is that of peer discussion, described by Crouch and Mazur [3]. The peer discussion procedure is, broadly: question presented; students vote; discuss in small groups; vote again; answers revealed; teacher-led whole group discussion (if needed). This way the peer discussion ought to cause the second votes to converge on the correct answer. Finding the answers through peer discussion rather than via a lecturer presented worked solution, the students ought to be encouraged into more active and deep learning. This method was considered but it was felt this would take more time than was available in the classes.

Initial trials with the TurningPoint system had shown it to be stable and reliable. The lecturer was provided with training and support in using the system to create questions. The technology was used in nine fortnightly examples classes over two semesters in the 2009/10 academic year. Students were told, in classes and via the module webpage,that the quizzes were to be based on current lectures. The message given via the module webpage was: "Keeping up to date with the lecture material, exercise sheets and assignments will be good preparation for [the quizzes]." In each session, students were provided with five questions on paper and given 15 minutes to work on these. After this, questions were displayed on screen and responses collected. The audience response distribution and correct answer were shown in class for each question and full worked solutions were made available via the module webpage. The lecturer wanted an evaluation, focused on whether the use of technology was having the desired effect on student engagement with module materials between lectures.

2. Method

An evaluation based on academic performance ([4], [5]) was ruled out due to natural variation in module marks year-on-year. The aim of the technology intervention was to influence student behaviour around review of lecture notes, so students were surveyed via a questionnaire on their behaviour in relation to the use of the technology.

Students were asked on a five-point Likert scale from "1 – Strongly disagree" to "5 – Strongly agree" to indicate the level of their agreement with the following three statements:

Q1: I like to keep up to date with understanding my lecture notes

Q2: I have been able to keep up to date with my [this module] lectures

Q3: I have been able to keep up to date with my other maths lectures this year.

They were asked other questions as well related to their views on different forms of assessment the analysis of which is not covered here (see the Appendix for the complete questionnaire). The students were then asked to rate their approach to the audience response system questions:

When answering voting response system questions using the voting response system, please place yourself with a tick on the following scale:

I think carefully about the questions asked				I don't think, I just choose answers at random
☐	☐	☐	☐	☐

Figure 2: Question re the students' approach

Finally students were asked to indicate their responses to four scenarios (S1 – S4): if they thought they knew the answer to a question or if they had guessed; and for each of these, if their answer was correct or incorrect. They were asked to select all that applied from the following responses: Feel pleased with myself; Look at the marked solutions; Discuss the question with friends; Do nothing; Work through the question again; Look at my lecture notes; Other (please specify).

E.g. Scenario 1: If I thought I knew the answer:

If I was correct, I would (tick all that apply):

- ☐ Feel pleased with myself
- ☐ Look at the marked solutions
- ☐ Discuss the question with friends
- ☐ Do nothing
- ☐ Work through the question again
- ☐ Look at my lecture notes
- ☐ Other - please specify:

3. Results

The questionnaire was administered in the final audience response system session, which had approximately 100 students in attendance. 38 completed questionnaires were returned. Initial analysis suggested a split in behaviour of the students into three groups according to their approach to answering questions using the electronic voting system – see figure 2. Nine students answered "1" (group 1); 18 students answered "2" (group 2); and 11 students answered "3", "4" or "5" (group 3). Results are given for all students and for these three groups. Tables 2-4 give the mean, mode and standard deviation (SD) for questions Q1-Q3.

Group	Mean	Mode	SD
1	3.89	4	0.60
2	3.83	4	0.51
3	3.91	4	0.70
All	3.87	4	0.58

Table 2: Descriptive statistics of responses to Q1: "I like to keep up to date with understanding my lecture notes".

Group	Mean	Mode	SD
1	3.78	4	0.83
2	3.56	4	0.78
3	2.55	2	1.13
All	3.32	4	1.02

Table 3: Descriptive statistics of responses to Q2: "I have been able to keep up to date with my [this module] lectures".

Group	Mean	Mode	SD
1	3.78	4	0.67
2	3.17	3	0.79
3	2.91	3	0.94
All	3.24	4	0.85

Table 4: Descriptive statistics of responses to Q3: "I have been able to keep up to date with my other maths lectures this year".

Tables 5-8 give the number of students who selected each option for the four scenarios when they saw which was the correct answer.

Group	Size of group	Feel pleased with myself	Discuss the question with friends	Work through the question again	Look at the marked solutions	Do nothing	Look at my lecture notes	Other
1	9	7	4	0	1	1	1	0
2	18	17	3	0	1	1	1	0
3	11	11	4	0	1	2	0	0
All	38	35	11	0	3	4	2	0

Table 5 – Scenario 1: Answers selected for "If I thought I knew the answer, If I was correct, I would..."

Group	Size of group	Feel pleased with myself	Discuss the question with friends	Work through the question again	Look at the marked solutions	Do nothing	Look at my lecture notes	Other
1	9	0	3	4	7	0	2	0
2	18	0	13	7	13	0	6	0
3	11	0	6	1	2	6	2	1*
All	38	0	22	12	22	6	10	1

* other: "worry"

Table 6 – Scenario 2: Answers selected for "If I thought I knew the answer, If I was incorrect, I would..."

Group	Size of group	Feel pleased with myself	Discuss the question with friends	Work through the question again	Look at the marked solutions	Do nothing	Look at my lecture notes	Other
1	9	2	4	2	7	0	1	0
2	18	6	6	6	8	1	4	0
3	11	7	3	1	1	5	0	0
All	38	15	13	9	16	6	5	0

Table 7 – Scenario 3: Answers selected for "If I had guessed an answer, If I was correct, I would..."

Group	Size of group	Feel pleased with myself	Discuss the question with friends	Work through the question again	Look at the marked solutions	Do nothing	Look at my lecture notes	Other
1	9	0	3	3	6	0	2	0
2	18	0	9	5	13	1	5	0
3	11	1	4	1	1	7	1	0
All	38	1	16	9	20	8	8	0

Table 8 – Scenario 4: Answers selected for "If I had guessed an answer, If I was incorrect, I would…"

Table 9 shows the numbers of all students and of students in each of the three groups who took some remedial action (discussing with friends, working through the problem again, looking at the marked solutions or reading lecture notes) on the outcomes of each of the four scenarios.

Group	Size of group	Knew & correct	Knew & incorrect	Guessed & correct	Guessed & incorrect
1	9	5	9	9	9
2	18	4	18	16	16
3	11	4	8	4	6
All	38	13	35	29	31

Table 9 – Across the scenarios: Number of students who took some remedial action in the four scenarios.

Table 10 gives the number of times each response was chosen across all four scenarios.

Group	Size of group	Feel pleased with myself	Discuss the question with friends	Work through the question again	Look at the marked solutions	Do nothing	Look at my lecture notes	Other
1	9	9	14	9	21	1	6	0
2	18	23	31	18	35	3	16	0
3	11	19	17	3	5	20	3	1
All	38	51	62	30	61	24	25	1

Table 10 – Responses chosen across all four scenarios.

4. Findings

All three groups were similar in their desire to keep up to date with their lecture notes. The extent to which they were able to do this, however, shows some variation between groups, with group 3 students in particular tending to give lower ratings to the questions about being able to keep up to date with notes in this and other modules.

Of the nine group 1 students, if they thought they knew the answer and were correct, five would nevertheless take some remedial action. In all other scenarios these students would all take remedial action.

Of the eighteen group 2 students, if they thought they knew the answer and were correct, four students would still take remedial action. If they thought they knew the answer and were incorrect, all eighteen would take remedial action. If they had guessed and were correct, sixteen would take action and if they had guessed and were incorrect, again sixteen would take action.

Of the eleven group 3 students, if they thought they knew the answer and were correct, four would still take remedial action. If they thought they knew the answer and were incorrect, eight would take action. If they guessed correctly, again four would take action and if they guessed incorrectly, six would take action.

The most popular remedial actions that would be taken in response to a question, whether known or guessed, correctly or incorrectly, were to discuss with friends and look at the marked solutions. About half as popular were working through the problem again and looking at lecture notes.

The group whose members least often would feel pleased with themselves was group 1. The group whose members would most often take no action in response to a question was group 3. Group 3 students would much more often discuss a question with friends than take any other remedial action.

5. Discussion

The students whose answer to how they approached the quizzes suggests that they were more engaged with them, exhibited different behaviour to those who indicated they were less engaged. A causal link between engagement and behaviour is not clear. Splitting the students into groups on their reported approach means we might characterise these groups. Those in group 1 are thinking carefully about all or most of their answers, those in group 2 are doing so for most of their answers and only guessing after they have thought about the question, while those in group 3 are more often choosing answers at random rather than trying to answer correctly. It is tempting to suggest that group 1 students are more able and group 3 students are less able, although the data collected do not include performance metrics of this kind.

That group 3 students are more likely to be choosing their answers at random makes it even more worrying that their response to guessing a correct answer was very rarely to take remedial action. Since the lecturer has recorded a correct response and not recorded whether the response was a guess, and the student does not take remedial action, these students could have gaps in their knowledge corresponding to the answers they guessed correctly which are masked all the way to the summative assessment. An approach where the questioning system asks whether the answer is a guess or asks for a score of confidence in the answer might provide useful feedback to the lecturer in this case.

Group 3 students do not take remedial action to the same extent as group 1 and 2 students, even though our characterisation suggests they may have a greater need to do so. It may be that the audience response method is not encouraging them to take such action, or it may be that they are encouraged to a limited extent and would take remedial action to an even lesser extent under another formative assessment method. An analysis comparing students using the audience response system with a cohort who are not, or comparing this cohort's behaviour in another module where the audience response system is not used, could provide useful evidence in this regard.

When they would take remedial action, the action that would be taken by group 3 students was most often to discuss the problem with friends. This is a negative result against the aim of encouraging engagement with the notes between lectures and indicates the possible advantage of peer instruction, particularly if group 3 students can be encouraged to speak with group 1 and 2 students.

Group 1 students gave very similar answers to the questions about keeping up to date with lecture notes in this and other modules, which suggests they would keep up to date with lecture notes in this module without intervention, so we might consider the intervention of the quizzes as not targeted at these students. Another result to deduce from these data is the extent to which students in group 1, and, to a lesser extent, groups 2 and 3, took remedial action even when the system confirmed for them that they had worked out the correct answer and we might hope this would suggest to them that they were at the required standard and did not need to take remedial action. This indicates that any optional materials or activity provided for the benefit of the less engaged students may well be taken up by the more engaged students, and raises a concern over whether the more engaged students might then be overloaded by too much optional intervention targeted for those who are less engaged.

These findings do not provide support for the described method of use of the audience response system when the aim is to encourage students to keep up with reading lecture notes during a module. There is some support for peer instruction, in that the least engaged students (the "least able"?) would tend to speak to their friends about the questions and, if this could be incorporated into the process, that may have advantages. There is also a note of caution for excess optional activities that might lead students to specific remedial action, in that some students (the "more able"?) might over-engage, unnecessarily, and risk overloading themselves.

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Endnotes

1. Note on terminology: The use of terminology in the literature is not very consistent. Simpson & Oliver [2] report use of: "Electronic Voting Systems", "Audience Response Systems", "Personal Response Systems", "Group Response Systems" and "Classroom Communication Systems". In personal conversation the authors have also heard "Student Response Systems". "Electronic voting" is a term also used in technology-enabled electoral polling and perhaps suggests the system is used only for 'opinion' questions when it certainly has no such limitation. "Classroom Communication System" is evocative of a desirable scenario but suggests more of the technology that the simple operation it performs. It is a system for collecting responses from individuals in a group; whether this develops "classroom communication" is dependent on the particular use. Here "audience response system" will be used as this describes well what the system does: it collects and collates responses.

Appendix

Questionnaire on assessment methods

Year: 1st or 2nd

Course, e.g. G100

Please indicate whether you agree or disagree with the following statements:

1. I like to keep up to date with understanding my lecture notes

Strongly disagree	Disagree	Neither agree nor disagree	Agree	Strongly agree
☐	☐	☐	☐	☐

2. I have been able to keep up to date with my G11APP lectures

Strongly disagree	Disagree	Neither agree nor disagree	Agree	Strongly agree
☐	☐	☐	☐	☐

3. I have been able to keep up to date with my other maths lectures this year

Strongly disagree	Disagree	Neither agree nor disagree	Agree	Strongly agree
☐	☐	☐	☐	☐

In the following questions,

U means assessment usually with feedback but which doesn't count towards the overall score (formative)

A means assessment which is marked and counts towards the module mark (summative)

Where applicable, please rate the following, thinking about **all** of your mathematics modules:

4. This form of assessment helps me to understand my notes

	Strongly disagree	Disagree	Neither agree nor disagree	Agree	Strongly agree
Coursework (A)	☐	☐	☐	☐	☐
Coursework (U)	☐	☐	☐	☐	☐
In class tests (A)	☐	☐	☐	☐	☐
In class tests (U)	☐	☐	☐	☐	☐
Voting device tests (U)	☐	☐	☐	☐	☐
Computer based tests (A)	☐	☐	☐	☐	☐
Computer based tests (U)	☐	☐	☐	☐	☐

5. This form of assessment clarifies areas I need to work on

	Strongly disagree	Disagree	Neither agree nor disagree	Agree	Strongly agree
Coursework (A)	☐	☐	☐	☐	☐
Coursework (U)	☐	☐	☐	☐	☐
In class tests (A)	☐	☐	☐	☐	☐
In class tests (U)	☐	☐	☐	☐	☐
Voting device tests (U)	☐	☐	☐	☐	☐
Computer based tests (A)	☐	☐	☐	☐	☐
Computer based tests (U)	☐	☐	☐	☐	☐

6. This form of assessment keeps me up-to-date with lectures

	Strongly disagree	Disagree	Neither agree nor disagree	Agree	Strongly agree
Coursework (A)	☐	☐	☐	☐	☐
Coursework (U)	☐	☐	☐	☐	☐
In class tests (A)	☐	☐	☐	☐	☐
In class tests (U)	☐	☐	☐	☐	☐
Voting device tests (U)	☐	☐	☐	☐	☐
Computer based tests (A)	☐	☐	☐	☐	☐
Computer based tests (U)	☐	☐	☐	☐	☐

7. This form of assessment is useful in helping me think about the mathematics

	Strongly disagree	Disagree	Neither agree nor disagree	Agree	Strongly agree
Coursework (A)	☐	☐	☐	☐	☐
Coursework (U)	☐	☐	☐	☐	☐
In class tests (A)	☐	☐	☐	☐	☐
In class tests (U)	☐	☐	☐	☐	☐
Voting device tests (U)	☐	☐	☐	☐	☐
Computer based tests (A)	☐	☐	☐	☐	☐
Computer based tests (U)	☐	☐	☐	☐	☐

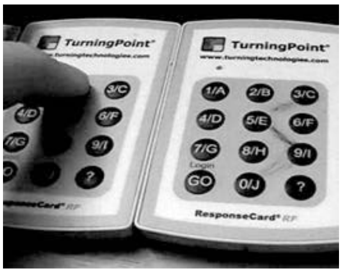
8. This form of assessment encourages me to seek help

	Strongly disagree	Disagree	Neither agree nor disagree	Agree	Strongly agree
Coursework (A)	☐	☐	☐	☐	☐
Coursework (U)	☐	☐	☐	☐	☐
In class tests (A)	☐	☐	☐	☐	☐
In class tests (U)	☐	☐	☐	☐	☐
Voting device tests (U)	☐	☐	☐	☐	☐
Computer based tests (A)	☐	☐	☐	☐	☐
Computer based tests (U)	☐	☐	☐	☐	☐

9. I like this form of assessment

	Strongly disagree	Disagree	Neither agree nor disagree	Agree	Strongly agree
Coursework (A)	☐	☐	☐	☐	☐
Coursework (U)	☐	☐	☐	☐	☐
In class tests (A)	☐	☐	☐	☐	☐
In class tests (U)	☐	☐	☐	☐	☐
Voting device tests (U)	☐	☐	☐	☐	☐
Computer based tests (A)	☐	☐	☐	☐	☐
Computer based tests (U)	☐	☐	☐	☐	☐

Voting response system



10. Approximately how many sessions have you attended using the voting response system?

☐

In the following, please think about your own behaviour in relation to the voting response system

11. When answering voting response system questions using the voting response system, please place yourself with a tick on the following scale:

I think carefully about the questions asked				I don't think, I just choose answers at random
☐	☐	☐	☐	☐

12. If you don't know the answer, please select which option you take:

- ☐ I pick one I think is most likely to be correct
- ☐ I don't answer the question
- ☐ I select an answer at random

13. If I thought I knew the answer:

If I was correct, I would (tick all that apply):

- ☐ Feel pleased with myself
- ☐ Look at the marked solutions
- ☐ Discuss the question with friends
- ☐ Do nothing
- ☐ Work through the question again
- ☐ Look at my lecture notes
- ☐ Other - please specify:

If I was incorrect, I would (tick all that apply):

- ☐ Feel pleased with myself
- ☐ Look at the marked solutions
- ☐ Discuss the question with friends
- ☐ Do nothing
- ☐ Work through the question again
- ☐ Look at my lecture notes
- ☐ Other - please specify:

14. If I had guessed an answer:

If I was correct, I would (tick all that apply):

☐ Feel pleased with myself	☐ Look at the marked solutions
☐ Discuss the question with friends	☐ Do nothing
☐ Work through the question again	☐ Look at my lecture notes
☐ Other - please specify:	

If I was incorrect, I would (tick all that apply):

☐ Feel pleased with myself	☐ Look at the marked solutions
☐ Discuss the question with friends	☐ Do nothing
☐ Work through the question again	☐ Look at my lecture notes
☐ Other - please specify:	

15. Please use this space and overleaf if required to give any further comments or feedback on the voting response system:

Thank you for your time in completing this questionnaire

Please return to Sally Barton, C2b Pope (or via any mathematics secretary).

Using history in mathematics teaching – some open education resources for the future

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Abstract

A recent collaborative project produced both audio- and text-based resources on the history of mathematics to support the teaching of mathematics. The benefits of using the history of mathematics as a motivational pedagogic tool in the undergraduate curriculum are discussed, with specific reference to the use of MP3 formats as a technique to support different learning styles and future modes of learning. The deliverables produced by the project are discussed.

1. Background: history and teaching mathematics

Mathematics is usually, and of course correctly, presented "ready-made" to students, with techniques and applications presented systematically and in logical order. However, like any other academic subject, mathematics has a history which is rich in astonishing breakthroughs, false starts, mis-attributions confusions and dead ends. This history gives a narrative and human context which adds colour and context to the discipline.

While some students may be attracted to mathematics by the apparently impersonal nature of the subject, the experience of the authors is that many students are engaged by historical information. Learning about the history of the subject shows that it is living and changing rather than being a fixed set of results known forever. Understanding that great mathematicians of the past struggled and sometimes made mistakes helps learners appreciate that errors are part of doing mathematics. For some, the human stories make mathematics exciting. For the potential research mathematician, becoming a member of this community includes buying into the stories mathematicians tell about the history of their subject. The fact that some of these stories are mythical makes them no less important in a shared mathematical culture [1].

The authors have noted in their own teaching that motivating students to the study of a new topic in mathematics can often be helped by setting the subject in a historical context. Indeed, it has been shown by Hagerty *et al* [2] that "the inclusion of historical modules caused positive changes in mathematical communication, student achievement and attitudes". This echoes the views of many other academics and educators at all levels from primary to higher education and teacher training [3],[4],[5],[6],[7]. In England the National Curriculum for Key Stage 3 Mathematics includes "the rich historical and cultural roots of mathematics" [8].

The authors' experience of teaching first year undergraduates is that presenting brief accounts of the history of the subject can be a valuable tool for another reason. Students start mathematics degrees at Universities like Greenwich with very diverse mathematical backgrounds: their previous study may or may not have included A (or AS) level Further Mathematics, they may instead have taken the International Baccalaureate, they may have studied Scottish qualifications or they may be international students with a completely different background. For mature students, it may be some years since they last studied mathematics. Similarly, for engineering students at the University of

Ulster, there can be a wide range of mathematical knowledge and ability amongst freshers, with some students being surprised and concerned at having to study more mathematics. Inevitably much first year material will be new to some students but very familiar to others, and presenting the historical background to the subject can provide new *and* stimulating material for those who are not being stretched by the technical content, without intimidating those for whom the main material is either new or barely remembered from distant schooldays.

Setting historical context can motivate and enthuse learners, but it also enriches the curriculum, shows connections between different branches of the subject, and helps to produce students with a greater sense of the breadth and, what might be termed, the creative life of mathematics as a discipline. We believe that many mathematics lecturers in higher education would like to include historical elements in their courses but lack either the subject knowledge, the time to prepare such material, or in some cases the confidence to engage with history.

There are, of course, many valuable general resources on the history of mathematics, such as book-length histories of mathematics ([9], [10], [11]), source books ([12], [13]) and reference works [14]. On the web the MacTutor history of mathematics website created by E. F. Robertson and J.J. O'Connor is widely used by academics and students [15]. Resources like these provide a great deal of information but we feel that, for lecturers seeking to provide snippets of historical material while presenting mathematics to their students, there is a need for "bite-sized" resources specifically relating to the undergraduate curriculum. The MacTutor article on Newton, for example, runs to six close-packed pages, which would not easily fit into a lecture on calculus. As van Brummelen notes, "school teachers and educational associations demand easily digested 'sound bites' that may be inserted with little fuss into an existing curriculum" [16], and the requirements of those teaching mathematics to undergraduates are similar.

Increasingly, there are opportunities to exploit students' familiarity with multimedia technology for learning purposes. Today's students are used to accessing material in audio format – for many, their headphones are always in place (even when they are following lectures!). Podcasts such as Math/Maths [17] and Travels in a Mathematical World [18] have gathered a following amongst undergraduate mathematicians. One member of the project team (Bradshaw) provided a series of history of mathematics podcasts for the latter, which were favourably received by students. Students already use their mobile devices to support their learning in sophisticated and imaginative ways, and the importance of resources for mobile devices can only increase as these become ever more powerful and flexible.

2. Methodology

These considerations motivated the authors to initiate a project, which was generously supported by the HEA MSOR Network with a mini-project grant, to produce resources to help lecturers incorporate history of mathematics into their teaching of mathematics. The overall aim of the project was the creation of a set of short "stand-alone" two-page documents together with MP3 audio files on a range of topics from the history of mathematics to supplement, and help motivate the teaching of, mathematical topics covered in the undergraduate curriculum.

By undergraduate curriculum we mean both mathematics degree programmes, and relevant service teaching. We have provided resources to facilitate the inclusion of historical material in modules covering mathematical methods and ideas in both mathematics degree programmes and service teaching. The materials are not designed for use by academics teaching modules on mathematics degrees which are specifically on the history of mathematics, where a more complex analysis of the issues would be required. (There are around 20 such modules embedded within mathematics degree programmes in British universities.)

We use the term "reusable learning objects" (RLOs) to describe these documents and MP3 files as each is relatively brief, self contained and independent of each other, providing an efficient way for teachers to embed the materials into their lecture courses and supporting teaching websites.

The material is designed to support the teaching of mathematics: it is not intended for historians. While the material aims to be historically accurate, we aim to present simple accounts that are entertaining, readable, and put the mathematics our students are learning into context. They are not the place for detailed consideration of sources or careful analysis of opposing views on controversial topics. We hope that interested students may be led to more detailed historical sources and that some (like the authors) will develop a fascination for the history of mathematics which they will pursue in more rigorous historical ways. But the purpose of these RLOs is to help students learn mathematics, and the history has to be presented with that end in view.

As part of the project the project team organised a one-day event run jointly under the auspices of the MSOR Network and the British Society for the History of Mathematics (BSHM). Entitled *The History of Mathematics in the Undergraduate Curriculum*, this event took place at the University of Greenwich on the 30th March 2010. Amongst those attending were recognised leaders in the field of history of mathematics and academics who teach the subject, or use history in teaching mathematics, at various universities in the UK and Ireland. This conference offered an overview of how the history of mathematics can be used, and identified some of the opportunities and issues. The feedback from like-minded practitioners was encouraging and had a strong influence on the development of the resources.

3. The RLOs: Benefits and Issues

Twenty RLOs have been produced, all of which are available on the project web pages [19]. These are "branded" with a logo designed by a mathematics undergraduate at Greenwich, Adam Sebestyen (Figure 1). They aim to provide support for teaching many different parts of the undergraduate curriculum. Topics such as "Mathematical Notation", and "π" present general background. "Archimedes" and "Islamic Mathematics" present mathematicians of different cultures. Core material is addressed by "Complex Numbers" and "Newton" and "Maclaurin and Taylor" support the teaching of calculus. "Euler", "Fourier" and "Laplace" present overviews of mathematicians whose names will occur during a mathematics degree in many different contexts. "Non-Euclidean Geometry", "Florence Nightingale", "Gödel's Incompleteness Theorems", "Galois", "Einstein and Relativity" and "The Schrödinger Equation" cover more specialist topics. The misconception that mathematics was all done in the past is addressed by "The Logistic Map", showing the recent development of Chaos Theory.

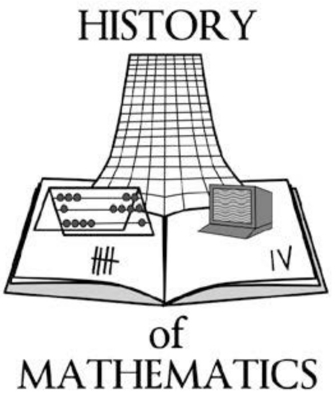

Figure 1: Project Logo

Our recent experience is that some mathematics undergraduates are hostile to the computing elements of the curriculum. We are often asked by students, whose experience of computing in school has not enthused them, why they need to acquire these skills. Our RLOs "From Beans to Bytes" and "Maskelyne and Comrie" attempt to address these negative attitudes by showing the long history of mathematicians turning to automatic calculation.

The text files are made available to be used in different ways. They provide a ready-made resource for a tutor to provide as background material for interested students. They provide a brief summary of historical context for a tutor who might adapt the contents in their own lecture material. Alternatively they could be cut and pasted directly into lecture notes.

The material is also available in audio format as MP3 files. These can be downloaded by students who might wish to listen to the background material on their journey to or from University, or while they are working at their computer. Each audio file lasts between 6 and 8 minutes, which the authors consider to be a length which students will find they can use in various ways – as a quick break from doing mathematics, or to fill a bus journey or a walk across campus. The MP3 and text files also address students' different learning styles.

The authors considered providing Powerpoint slides as an additional resource for lecturers. Our conclusion was that this was impractical for a number of reasons. Powerpoint is by no means universally used as a tool for delivering mathematical content, so such a resource would be of limited value. Furthermore, different lecturers use Powerpoint in very different ways so it is unlikely that resources could be "dropped into" lecture material without considerable adaptation, which would defeat the purpose of the RLOs.

A number of issues arise regarding the RLOs. That there is no standard curriculum for undergraduate mathematics programmes obviously presents difficulty for a project like this. However there is a large amount of core material that will be covered on any mathematics degree – calculus and complex numbers, for example. Other RLOs may not be relevant for some undergraduates – for example, statistics and computing are core in some programmes but some undergraduates, particularly those on combined degrees, may not study these topics. While the RLOs on calculus and complex numbers will be relevant for engineering students learning mathematics, they will be less meaningful for business students, but the situation will be reversed for the statistics and computing material. Clearly we cannot expect that every RLO produced will be useful for every student, but the diversity of topics covered should mean that some are relevant.

The subjects are predominantly white, Western and male. While we have attempted to address this deficiency, only the RLOs on π, Fibonacci, mathematical notation and Islamic Mathematics feature mathematics from outside the Western tradition, and Florence Nightingale is the only female mathematician featured. Sadly, we feel that this lack of diversity reflects the mathematics curriculum, which these resources are intended to support. Future additions might include figures like Emmy Noether and Ramanujan, who are important figures in mathematical culture but whose mathematics may not be encountered by most undergraduates.

While we have tried to cater for mobile devices by creating audio files as well as texts, we have not attempted to produce full multimedia presentations incorporating large amounts of visual images or video. These would be very time-consuming to produce, whether the visual elements were created for the purpose or were sourced from existing material, and would present copyright issues. (The images used to illustrate the text files are all out of copyright and sourced from Wikimedia Commons [20] to avoid these complications.)

The question of dissemination is vital, since the intention of the project is that these resources should be widely available. They are being promoted through conferences such as that at Greenwich mentioned above and the annual 2010 MSOR Network conference. The authors are also promoting them through professional bodies [21] and by posting to the MATHSHEADS email list.

4. Evidence of Success

Feedback from undergraduates who have used early versions of these materials has been very positive. Comments include:

- "*I want to do maths because I am inspired by knowing about the people behind it*"- First year undergraduate 2010. Module: Mathematical Technology and Thinking, B.Sc. Mathematics, Greenwich.
- "*A quick and easy educational tool.*" First year undergraduate 2011. Module: Mathematical Technology and Thinking, B.Sc. Mathematics, Greenwich.
- Listed as one of the three best things about the module: "*The extra information about people like Newton*." - First year undergraduate 2010. Module: Introductory Mathematics, B.Sc. Civil Engineering, Ulster.

We note that in all these cases the comments were made as part of the overall module evaluation process: the students were not specifically prompted for their views on the history of mathematics materials which had been embedded in the lecture notes. The fact that students drew positive attention to the materials 'unprovoked', as it were, is itself encouraging. Further feedback will be obtained in future from students and tutors, and a hyperlink has been added to the project web pages inviting feedback from users.

5. Reflections

This project has attempted to address the needs of future learners of mathematics. While we will not know for some time the extent of the uptake for our resources, the work has identified a number of issues about teaching mathematics in the future. One is the need to present learning materials in a variety of formats to take advantage of new technologies, and the changes in learning patterns resulting from the use of mobile devices. These will present challenges for those teaching mathematics in higher education. A second is the way in which these technologies facilitate the use of material like this to motivate the study of mathematics, and the possibility that different students will benefit from different motivating material. In the brave new world of UK higher education post-2012, the need to maintain the enthusiasm of learners will be paramount. Resources which complement traditional curriculum material offer an opportunity to do this. For some learners these resources might relate to the history of the subject; for others, resources connecting the course material to applications, showing how it can be used in the workplace, or connecting it to other branches of mathematics, might be more motivational. This project may suggest a way forward which has wider application than the historical context of mathematics.

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Does MapleTA produce Constructively Aligned Maths assessment?

Peter R Crompton

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Abstract

MapleTA is an online assessment package from Maplesoft, built on top of the computer algebra package, Maple, which can be used to assess mathematics as mathematics, provided that a student can be trained to enter their mathematical solutions into MapleTA in Maple syntax. We analyse the assessment performance of 312 first year undergraduate Maths students at the University of Leeds taking the computer algebra module "Basic Computing for Mathematics". In particular, we focus on the effect that self-grading during a MapleTA assessment has on grade performance, and whether this grade is correlated with a student's ability to apply Maple as a black box solver to obtain final solutions of past exam papers in conventionally taught first year Maths modules.

1. Introduction

It has been reported that computer algebra based online assessment, using the computer algebra package Maple, can be used to assess higher mathematical skills [1], and that this assessment design can form an important role in stimulating learning by providing an opportunity for students to create instances of objects for themselves. Although highlighted as important in the Quality Assurance Agency MSOR '07 benchmark statement [2], an obvious danger with introducing students to black box computer algebra solvers, such as Maple, is that this could undermine the development of their basic algebra and arithmetic skills. At the same time, generic to all forms of online assessment, computer algebra based assessment presents the opportunity to provide rapid feedback to students and formative assessment [3]. In this study we have aimed to quantify if such a combination can be an effective way to deliver constructively aligned Maths assessment [4]: a learning environment defined maximize the opportunity for students to create instances of objects for themselves, and to support high level learning. We conclude that assessing a computer algebra course via MapleTA is an effective way to design a constructively aligned assessment for past exam revision in conventionally taught first year undergraduate Maths modules.

2. Cohort

We have assessed three mixed cohorts of first year single and joint honours Maths degree students at the University of Leeds during 2008/09 and 2009/10 using MapleTA. The first cohort consisted of 66 students, the second 222 and the third 24, giving a total of 312 first year single and joint honours Maths students included in this study. The proportion of single versus joint honours Maths students was, in total, roughly 1:1. Each cohort was assessed using MapleTA whilst enrolled on the 5 credit computer algebra module "Basic Computing for Mathematics". This University of Leeds module is designed to introduce first year Maths students to the Maple syntax required for basic problem solving in Calculus, Linear Algebra and Analysis, and is pitched at a level of mathematical difficulty comparable to the Maths examples and exercises of their broader first year Maths programme on these topics.

Although there is a degree of mathematical problem solving involved in using a computer algebra package such as Maple, in essence, Maple performs the role of a black box solver for calculations. The challenge in Maple syntax assessment design is therefore to acknowledge the calculation elements which can be performed more expeditiously using a black box design, and to incorporate these black box elements as explicit algebraic elements into the question design [5]. For example, in a matrix multiplication exercise one could use upper triangular or Hadamard matrices, for which it is implicitly more expeditious to use mental arithmetic than a black box approach to evaluate the matrix product [6].

Each of the three cohorts in "Basic Computing for Mathematics" was assessed via 4 assessments delivered via MapleTA, and one tutor marked assessment. Each of the assessments lasted one week and was supported by two hours of Computing Lab sessions with attendant postgraduate demonstrators, and by one hour of accompanying lecture. On average, each student spent approximately 4 hours working on each of the 5 assessments. The fifth tutor-marked assessment consisted of an exercise designed to apply the Maple the students had learnt during the "Basic Computing for Mathematics" module to answer past exam paper questions on the topics of Calculus, Linear Algebra and Analysis. These past exam question were taken from each of the separate first year modules on these topics. The format of the fifth assessment was that the students were given a question directly taken from these past first year exam papers (or were given a question on similar material in this question format), and were asked to produce a set of final solutions for these questions using Maple. In essence, the fifth assessment was designed for this study as an independent check on the level of knowledge of Maple syntax developed during the module and its relation to the mathematical content of the broader first year Maths degree program. The students reflected in their module feedback surveys that assessment 5 was a useful preparation for their past exam paper revision for Calculus, Linear Algebra and Analysis, since in order to cross-check their Maple syntax in many instances they were required to perform the calculations explicitly.

3. MapleTA

MapleTA is an online assessment package produced by Maplesoft. Unlike online assessment packages such as QuestionMark, the functionality of MapleTA is built on top of a computer algebra package, Maple. Therefore, in addition to the usual javascripted Boolean matches to an existing solution set (the model answer) MapleTA is able to provide a secondary Boolean matching to the underlying algebraic logic of that solution set. In effect, MapleTA is able to mark mathematics as mathematics [5], rather than as a character string or as elements of a Multiple Choice Question solution set. As an example, if a student were asked to evaluate the indefinite integral of sin(2x)cos(x) any of the following three expressions in Table 1 below would be intrinsically marked by MapleTA as equivalent correct solutions. There are clear advantages and disadvantages to this form of online assessment for mathematics students. Perhaps the most obvious disadvantage is that neither of the first two rows in Table 1 are actually the formally correct solution to this indefinite integral (both should come with +c integration constant, which is not generated in the output of the underlying computer algebra package, Maple).

MapleTA input	Mathematically equivalent expression
-1/6*cos(3*x)-1/2*cos(x)	$-\frac{1}{6}\cos(3x) \ -\frac{1}{2}\cos(x)$
-2/3*cos(x)^3	$-\frac{2}{3}\mathit{cos}^3(x)$
int(sin(2*x)*cos(x),x)	$\int \sin(2x)\,\cos(x)\,dx$

Table 1: Input to MapleTA for the solution to $\int \sin(2x)\,\cos(x)\,dx$

A second disadvantage for the mathematics student is that a certain basic level of knowledge of Maple syntax is required in order to enter solutions of this form into MapleTA, and in particular, an understanding of the precedence of operations (*+- /) as they appear in the use of a basic calculator. Whilst the algebraic logic of Maple is robust, in recent years Maplesoft have reportedly lost market share due to some of the awkwardness of Maple syntax. So, in the above example, whilst -2/3*cos(x)^3 would be accepted by MapleTA as a correct solution, -2/3*cos^3(x) would not, even though the latter is perhaps the more natural syntax for the precedence of the power operation for a mathematics student. This awkwardness problem of Maple syntax is somewhat overcome, however, by the MapleTA preview tool which will allow a student to display the Maple syntax in the left hand column of Table 1 as the mathematical expressions given in the right hand column of Table 1. Finally, perhaps the most serious disadvantage of MapleTA for the assessment of mathematics is that in the above example it is not necessary for a student to evaluate the indefinite integral – the expression in the third row can be inputted into MapleTA [3], which is the instruction for Maple to evaluate the indefinite integral itself!

Despite stressing these three failings, MapleTA clearly performs a clever task in marking algebraically equivalent statements as correct. It should not be understated that, with large first year and foundation year cohorts, online mathematics assessment has the potential to significantly reduce the postgraduate demonstrator and staff marking assessment overheads, both in terms of man hours and direct costs. MapleTA has also found a large market in the US (and now Europe) for Placement Testing (streaming to specific degree programs) at the foundation/first year level, where it appears to be successful in combating high drop-out rates through its formative assessment design [7][8].

4. Self-grading

We refer to self-grading in this article as the activity defined by a student who accessed their MapleTA grade during an assessment in order to obtain feedback. In this study we were interested in finding out the degree to which self-grading during a MapleTA assessment can influence the final grades of an assessment, and the degree to which self-grading during a MapleTA assessment can affect learning on a given Maths topic. We chose to set up the assessments with unlimited question attempts, but to limit the feedback displayed to each student to a tick/cross grade display along with the students most recently entered solution for each assessment question. Each of our 4 MapleTA assessments also consisted of 4 questions.

One of our main motivations for setting up our MapleTA assessments with this particular format was in response to the module feedback survey results we received which overwhelmingly cited the students' failure to be able to identify problems with Maple syntax as the main source of error contributing to their assessment grades [9]. Our aim was therefore to minimize the syntax problems associated with learning Maple by allowing the students to use the MapleTA online assessment package as an effective debugging tool for Maple syntax. Our intention was that one quick grading would quickly allow a student to identify that MapleTA will recognize a difference between -2/3*cos(x)^3 and -2/3*cos^3(x), thus allowing the student to focus on the mathematical questions associated with finding the solution to the evaluation of an indefinite integral for sin(2x)cos(x), rather than the Maple syntax, in this example.

5. Statistical Analysis of Self-grading

We have recorded each of the combined cohort of 312 students' grades, via MapleTA, for each of their (unlimited) attempts at our 4 MapleTA assessments for the University of Leeds first year computer algebra module "Basic Computing for Mathematics". We have then performed two basic statistical analyses on this data in order to examine the significance that self-grading has had on their grade for each assessment (where self-grading is defined as the activity of a student who accessed their MapleTA grade during an assessment in order to obtain feedback), and also to determine the effect that self-grading has had on the learning of a given Maths topic.

In Figure 1 we present a boxplot of our recorded MapleTA data, where the length of the boxes indicates the interquartile range of grade for a given number of self-gradings, the whiskers indicate the spread of grade values for a given number of self-gradings, and the width of each box is proportional to the square root of the number of students with that a particular number of self-gradings. As we would expect, there is a sharp rise in

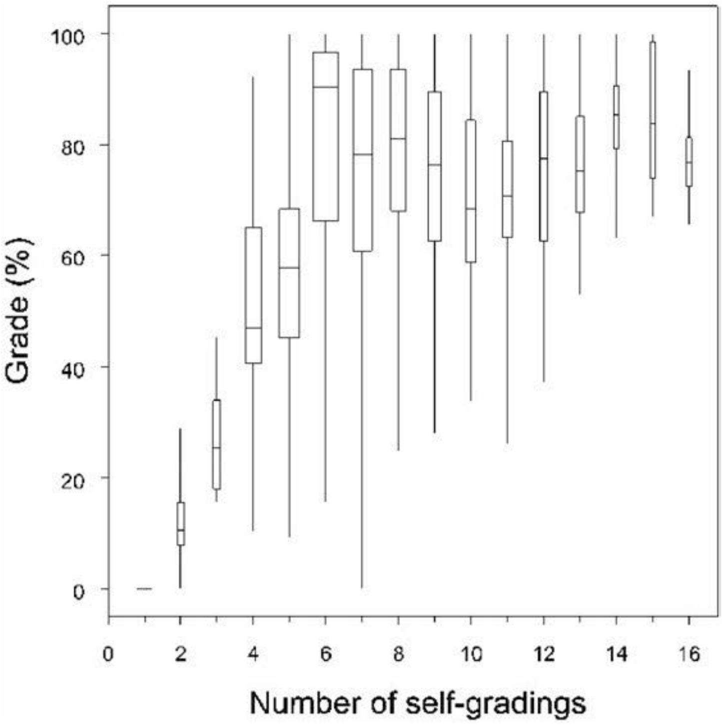

Figure 1: Assessment grade versus number of self-gradings

the grades (from 0% to 50%) for students with 0 – 4 self-gradings, which corresponds to students who made one complete assessment attempt. However, it also appears that this trend continues beyond 4 self-gradings such that a student who self-graded 6 or more times received almost twice the grade of a student who self-graded 4 times. Moreover, the spread of grade values associated with 4 self-gradings and 6 self-gradings is similar, which indicates that the distribution of the grades of students with 6 self-gradings is skewed in favour of higher grades. Similarly, the distribution of student grades remains skewed in favour of higher grades for tallies of 6 – 12 self-gradings. This indicates that the average grade is higher than would be expected from a normal distribution of the student abilities, which indicates that weaker students were more likely to benefit from self-grading.

To interpret this data further we performed a one-way ANOVA analysis using the self-grading count as a factor. From the ANOVA, we got an F statistic of 48.09 on 32 and 1247 degrees of freedom, with a corresponding p value of <0.001. Since this p value is very small, there is very strong evidence that the average assessment grade depends on the number of self-gradings generated by the student.

To analyse the effect that self-grading had on the students learning of a given Maths topic we used a General Linear Regression model with stepwise regression to identify the important explanatory variables for finding a fit to the total assessment grade of each student. This total assessment grade consisted of the sum of each of the 4 MapleTA assessments and the fifth tutor marked assessment, weighted equally. As explanatory variables we chose the number of self-gradings generated by the student for each question within each of the 4 MapleTA assessments, and the grade of each student for each question within each of the 4 MapleTA assessments (i.e. we excluded the grade of the fifth tutor marked assessment to construct a contrast [10]).

For each of the 3 cohorts, over 85% of the variance of the total assessment grade could be accounted for by a model which included the grades for the 30% of the assessment questions with the lowest average grade and lowest number of attempts (the hardest questions) and number of self-gradings for the 20% of the assessment questions with the highest average grade and highest number of attempts (the easiest questions). This regression had an F statistic of 22.04 on 34 and 311 degrees of freedom, with a corresponding p value of <0.001, which again indicates a good model. Comparing with Figure 1 this result is not unexpected, since the grade distributions of students either failing to complete one attempt of an assessment, or successfully completing multiple attempts (far right), are the least skewed. Therefore, these two groups of students are the most likely to contribute to the linear regression model. We can conclude that the average total assessment grade is determined by the weakest and strongest students, and that the effect of self-grading in MapleTA assessments on the students' learning of a given Maths topic is to improve the performance of a student with an average grade.

6. Conclusions

We have performed a statistical analysis of the correlations between the number of self-gradings in a MapleTA online Maths assessment (defined as the activity of a student who accessed their MapleTA grade during an assessment in order to obtain feedback) and the grades received by the student for that assessment. The grade feedback displayed to each student was a tick/cross question display, along with the students most recently entered solution for each assessment question. Part of our motivation for using this assessment design was to eliminate the need for students to struggle with inputting Maple syntax into MapleTA, which was highlighted as a concern on their student feedback surveys, thus allowing them to focus on the mathematical problem solving component of the assessment. Our one-way ANOVA results indicate that average student grade performance is enhanced by self-grading, in some instances more than doubling the average grade as a function of the number of self-grades (see Figure 1). We have also investigated the effect of self-grading in MapleTA assessments on the student learning of a given Maths topic, by performing a contrast regression analysis [10] on an independent tutor marked assessment given on the same Maths topic. These results confirm that the average student performance is the most strongly affected by self-grading, where the strongest students grade distributions have essentially been maxed out by the effect of self-grading whereas the weakest students have not engaged. With regards to the constructive alignment of Maths assessment, we can therefore conclude that MapleTA can provide a useful function in helping to prepare students for examination in conventionally taught first year undergraduate Maths modules.

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How do students deal with difficulties in mathematics?

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Abstract

We report on a study carried out by the Mathematics Department at the National University of Ireland Maynooth to determine why students do or do not engage with mathematics support. Initial interviews were conducted with students who had failed first year. This paper gives preliminary findings from interviews with a second group of students who had passed first year. Students were chosen who had similar mathematical backgrounds to the first group and who had all engaged with mathematics. The students' mathematical backgrounds do not appear to be the only major factor in determining engagement. We found that both groups experienced similar difficulties and problems. However, the second group had several different strategies or coping mechanisms to enable them to get through. We compare the two groups and will discuss some of these coping mechanisms in detail.

1. Introduction

This paper forms part of a study at National University of Ireland Maynooth (NUIM) into why students do or do not avail of the many mathematical supports in place if they experience difficulties. A previous statistical analysis showed that students who engaged with these supports were more likely to succeed than those who did not [1]. An earlier part of this study focused on a group of students who had not engaged with mathematics. This paper will focus on a group of students who had engaged with mathematics and will compare and contrast these students with the original group. The students have similar mathematical backgrounds and consequently the main aim of this study is to discover the real reasons students do not engage with mathematics.

Other authors have found that the fear of showing a lack of knowledge or ability negatively impacts on students' willingness to ask questions [2]. In [3] we found that factors important in examining non-engagement with mathematics included the demoralising effect of failing first semester examinations, the anonymity of large classes, and to a lesser extent the lack of awareness of support services. Many of these factors were also identified in a study of students at Loughborough University [4]. As a result of these findings [3] we decided to conduct a study of students with similar mathematical backgrounds to the first group but had engaged with mathematics. A preliminary analysis suggests that although these students encounter many of the same difficulties as the group of students who had not engaged, the majority have some plan or strategy to overcome these difficulties. They mention friends and working with peers whereas the non-engagers rarely mentioned these supports. The engaging students appear to be very aware of their own learning style and what works for them. The main focus of this paper will be the varying strategies used by these students.

2. Methodology

Thirty-nine students who were repeating first year mathematics modules were identified in September 2009. These students were contacted and asked to take part in this study. Twelve students agreed to fill out a short questionnaire and seven of these agreed to be interviewed. Coincidentally, the seven students who agreed to do interviews were all students who had not actively engaged with mathematics or mathematics support. The supports consist of small-group tutorials and homeworks which are corrected and handed back in the tutorial every week. In addition, a diagnostic test is administered to all incoming first years. Students who score 20 or less out of a possible 60 marks are deemed to have failed the test and they are then registered for online courses which have been designed to help students with weak mathematical backgrounds. The Department of Mathematics also runs a very successful Mathematics Support Centre (MSC). The first group of students generally had poor attendance at lectures and tutorials, poor submission rates with respect to assignments and had rarely attended the MSC. The group comprised of four male Science students, two female Finance students and one (mature) female Arts student. Mathematics is a compulsory subject in first year for Science and Finance students.

In February 2010, we decided to compare the first group with students who had engaged with mathematics. We considered students who had attended the MSC at least five times and had passed their first year exams. We also split these students into two categories which corresponded to the mathematical background of the first group. The first category comprised of students who had taken Leaving Certificate (LC) higher level mathematics or who had scored an A grade in ordinary level (OL) combined with a pass in the diagnostic test. The second category comprised of students with a LC grade of B or lower in OL mathematics and a fail on the diagnostic test. The Leaving Certificate is the final exam at the end of second level education in Ireland, all students take mathematics. Ten students were contacted, nine students responded and were interviewed. They also filled in a short questionnaire. They generally had good attendance at lectures and tutorials and had submitted the majority of assignments. The group consisted of two male Arts students (one a mature student), two female Arts students, two male Science students (one a mature student) and three female Science students.

The interviews were conducted by the first author. Each interview lasted for approximately forty minutes. The questions were open-ended and concerned the student’s mathematical education prior to enrolling at NUIM as well as their experiences of mathematics in the first year of their degree. They were questioned on their experiences of lectures, tutorials, assignments and the MSC. The interviews were the same for both groups but with the addition of a detailed section on the services provided by the MSC for the second group.

The interviews were transcribed by the first author. All three authors coded the transcriptions using Grounded Theory [5] and the codes were then compared. Pseudonyms were used to protect the students’ identities.

3. Results

A preliminary analysis of the data has been carried out to date. It was apparent from the initial analysis of the first group [3] that they had almost exclusively not engaged with mathematics. Analysis showed that students were often not aware they had a problem or were unwilling to admit it (to themselves or others) until it was too late. Students were also reluctant to ask for help and feared embarrassment. Mathematical background is an important factor in determining grades but some students from our first group had a good LC grade and had passed the diagnostic test. We would not have labelled them as “at-risk” students. However, it was apparent that other factors also played a major part. Subsequently we will present some of the other possible reasons why the second group engaged and succeeded with mathematics.

3.1 Similarities

It was apparent that both groups encounter similar difficulties and problems. The main category to emerge from the coding of the first group of interviews was that of fear or embarrassment. A more detailed analysis can be found in [3]. The fear category appeared to comprise of four separate but overlapping concepts, fear of failure, fear of the unknown, fear of being singled out and fear of showing a lack of knowledge or ability. Here, Jonathan from our first group discusses the MSC, an example of fear of showing a lack of knowledge or ability:

"I was actually really embarrassed and intimidated about going and saying listen guys I struggle horribly with maths."

Similarly, Janice from our second group shows the same kind of fear when discussing asking a lecturer a question:

"You'd be afraid to go to a lecturer, they're doing this for ages and they've got their PhDs…and you're going in and asking them about the domain and range…they're looking at you as if to say, "Ah come on now, we did this in the first lecture!"

3.2 Differences and Coping Mechanisms

Similarities amongst both groups are to be expected as both groups had similar mathematical backgrounds. Similarly, we would expect differences because students in the second group engaged and succeeded at mathematics.

We will outline how the attitudes and experiences of the first and second groups contrasted. This will give us some insight into the different mindset of the two groups. Then we will outline the main strategies or coping mechanisms that emerged from the analysis of the second group.

Two basic issues on which the groups differed were awareness of supports and attitudes towards supports. Interestingly all the students in our second group were much more aware of the supports available to them from the college and their peers. Students from our second group discuss their awareness of the MSC:

"Ciarán (MSC manager) came in…our first or second lecture and told us about it…the opening hours and I took them down straight away and then after the first week I said I had better go" (Janice)

"They were telling us the MSC was there before you actually started the course" (Susan)

Students in the first group did not seem to be as aware of the MSC. It may be the case that these students were already not engaging at the start of semester one. Craig from our first group speaking about the MSC drop-in centre:

"I was vaguely aware of it…I thought it was an extra class…I would have thought it was one hundred students just being lectured again"

Another interesting difference is in the attitude of our two groups. The first group are more negative towards certain aspects of mathematical support. To explain, we can see how students in both groups interpret the marking system of assignments. Students submit five questions and one is corrected at random. A typical response from the first group:

"Just randomly picking just one question and us not knowing is unfair" (Bob)

Whereas a student from our second group can see positives in this system:

"It is good that it shows you that you have to do every question, it makes you do the whole lot of them cause you might not do one and you get zero for it." (Liz)

Overall it is quite clear that the two groups have contrasting attitudes on several issues. The main difference is that the second group all seem to have some plan or strategy whereas our study of the first group revealed that the majority of them did not seem to have a coherent strategy in their approach to engaging with mathematics.

We will now give a breakdown of the main coping mechanisms that have emerged from the analysis to date. The first type of strategy is exemplified by three of our students, David, Joe and Andrew, who are all mature students. That strategy is to avail of almost all possible supports. Here Joe is asked about why he attends an extra tutorial available to his class group:

"I was coming in with such a low level. I was just gonna give myself every chance. Anything that was going I was gonna use it"

Interestingly, Andrew talks about how he knows when he had enough resources or supports to get him through. He does not allow himself to be overwhelmed and chooses to not avail of some supports. Here he discusses an online refresher course that covers basic topics:

> *"I did use that…it refreshed all the basic rules…for someone like me who had had a break from it (mathematics) for a long time…Towards the end of the semester, when we were a lot busier I found it more of an inconvenience than a help"*

Perhaps the reason for the mature attitude these students have to engagement is due to the fact that they are older and more experienced than a student who has recently left school. David discusses the possible advantage of being a little older coming to university when asked about asking questions in lectures:

> *"…maybe not in lectures cause I think people feel it's a bit more intimidating cause they can say, "…am I the only one thinking this?". But I do think being a little older has helped…I think maybe if I was 18, I would have been a bit more reluctant"*

What was also apparent was that some of our students favoured indirect strategies over direct strategies such as going to the MSC with a friend and allowing the friend or group of friends to ask the questions while they absorb the information. In addition to this students placed high value on the support of their peers. What is apparent is that they seek help from the sources they are comfortable with. It is also interesting that these students had similar fear and embarrassment issues to the first group but with this indirect approach they managed to circumvent it and get help. Andrea contrasts one-to-one help and group work in the MSC:

> *"If I ever went over on my own I hated it…felt kind of stupid, even though everyone's really nice…when I was in a group it was brilliant cause everyone would work out different pieces or something. I'd usually go with 4 or 5 people…when the tutor came over, they'd just explain it to everyone"*

The final type of strategy is a realisation that it was not absolutely necessary for them to attend fully or to understand fully all of the material to progress. This came as a surprise to us considering these students had engaged and succeeded with mathematics. This strategy was adopted by some of our group but by Shaun in particular. Shaun repeatedly expressed a preference for one-to-one tuition. He initially attended all his classes but soon began to realise that he was not gaining any extra understanding through that:

> *"I just realised all I was getting out of the lectures was notes and all I was ever doing was looking over the notes when I came out of it and I got the same understanding of it"*

Shaun instead would get the notes from a friend, work on them himself and then go to the MSC and ask for help there. On the issue of understanding Shaun talks about how he feels he has not enough time to go back and correct his understanding. He realises he does not need a full understanding to get through the year though. Here he talks about why some of his peers are failing and he is progressing he says:

> *"I find that a lot of people that are failing, they are going to the lectures and trying their best but they're just not 'smart studying', exam studying"*

It is clear that students who engage do not have a uniform strategy. Several different strategies have emerged and some of those were unexpected. Investigations into these are ongoing.

4. Further Work

We have already designed interventions as a result of our findings from the first group. These interventions were trialled in 2009/10 and are in operation again this year. They include a mentoring programme where students are offered the chance to come and talk to a sympathetic faculty member. We are also monitoring engagement and intervening with students who do not engage. We also designed a document for incoming first years explaining the differences between mathematics at second level and third level and the structures and supports in place to help them through.

We are also analysing 470 questionnaires completed by first year mathematics students last year. Students filled out a questionnaire at the beginning of semester one and the end of semester two. The questionnaire contained sections on confidence in one's mathematical ability, their perception of how useful mathematics was and also a section on study methods. We hope that with further analysis of both our transcripts and these questionnaires that we can design and implement more interventions that will help with the problem of student engagement.

5. Conclusion

From the preliminary research it is apparent that mathematical background is not the only major factor in predicting whether a student will engage and succeed with mathematics. From our analysis of the second group there is clearly some motivation, some desire that pushes them through and forces them to engage to some extent. We hope that further coding and analysis in conjunction with re-examining the first group will reveal exactly what this motivation is.

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Assessment for learning: Using Moodle quizzes in mathematics

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Abstract

The introduction of online, interactive computer marked assessments within a distance taught foundational mathematics module is discussed. The aim of the assessments is to encourage and support learning. The style of assessment used is described together with details of their development and how they are integrated within the module teaching strategy.

Data showing how the assessments were used by students on the first presentation of the module are given, together with some student feedback and issues encountered. Although the uptake of these assessments was not as great as hoped, the student experience has been generally positive.

1. Introduction

The Open University (OU) is the UK's leading provider of distance education, with students generally studying at home using provided resources and being supported by an allocated part-time tutor. In February 2010 the OU launched a new foundational mathematics module *MU123: Discovering Mathematics* to replace the previous offering. This 30 credit module introduces topics such as basic algebra, geometry, trigonometry, exponentials and statistics. It is offered twice per year with approximately 1,600 students per presentation.

A blended teaching approach is taken, with printed texts supplemented by interactive resources on the University's Virtual Learning Environment (VLE). The OU VLE is based on a customised version of the open source Moodle system [1]. The online resources include short video screencasts explaining some of the examples in the texts; interactive applications for investigating statistics, graphs and geometry; and extensive online assessments, both formative and summative.

In this paper, the rationale for including these online assessments is described, together with the particular style of assessment used and their development. Data demonstrating student usage during the first presentation of the module is presented, along with unsolicited comments from student discussion forums.

2. Online interactive computer marked assignments

2.1 Rationale

The main form of assessment within mathematics at the OU is the tutor marked assignment (TMA). A student (typically) hand writes solutions to posed problems which are then posted directly to their tutor for marking. The marking usually includes extensive individualised feedback and teaching comments. The script is returned to the student by post, via the University. Whilst providing excellent feedback and teaching, the delays inherent in this system can reduce the effectiveness of that feedback [2]. The use of electronic submission techniques, which have been adopted by much of the University and trialled in mathematics [3], can reduce these delays, yet teaching by this method still falls short of an interactive dialogue.

In many modules, TMAs have been supplemented with computer marked assignments (CMAs) where all students are given the same questions, answers are entered via a web-site and results are given after a submission deadline. Again, this does not permit immediate feedback to the student. Such feedback would be particularly valuable to distance learning students as it enables them to swiftly highlight any misunderstanding or misconceptions before they become too ingrained.

There are a variety of systems available that can support such interactions, both commercial and open source. In addition, many of these systems support the generation of randomised variants of individual questions, which both enables the student to practise as many examples as they feel they need, and allows different students to be given different questions to reduce opportunity for plagiarism in summative assignments. In recent years, the OU has started to make use of a number of these systems, in particular OpenMark [4], a system developed at the OU and now available as open source software, and the quiz system of the Moodle VLE, which is currently also maintained by the OU.

The aims of introducing such VLE-based assessments in *Discovering Mathematics* were to enable more effective learning by providing students with immediate feedback (including specific feedback to common errors) and providing students with sufficient opportunity to practise.

2.2 Implementation

A formative "Practice Quiz" was developed for each of the fourteen study units of the module. These are designed to enable students to check and consolidate their understanding of the content of the unit as they progress. To help students engage with the learning of mathematics as they use these quizzes, they are permitted up to three attempts at each question. After each incorrect attempt the student is given a graduated hint and in many cases a specific comment appropriate to their incorrect answer. The first hint given is a reference to an appropriate location in the associated study text, the second being more detailed guidance on how to attempt the question, possibly reminding them of a formula or technique that is required. After the final attempt, or indeed once the correct answer has been given, a full worked solution to the question is provided. This type of question behaviour is known as "Interactive with multiple tries" and has been developed within the Moodle quiz engine by the OU. It will be made available to the Moodle community as part of Moodle version 2.1.

A variety of different question types are used within the Practice Quizzes, including

- multiple choice: where one answer is to be selected from several possibilities;
- multiple response: where several answers are to be selected from a list of options;
- matching: where several answers are to be selected from a set of drop-down lists;
- numerical: where free-form numerical input is matched against target ranges;
- drag-and-drop: where items of text need to be selected and moved on the screen to their correct location within a statement or sentence;
- short answer: where free-form text is matched against answer templates.

The final type is mainly used for the input of fractions in a linear syntax.

All the questions offered are randomised, so that a student can attempt each quiz more than once (for practice and reinforcement), and will (probably) get a different set of questions each time. Currently, the OU VLE is unable to generate randomised questions on-the-fly, but does support the selection of a random question from a pre-generated pool. This randomised selection is used within the Practice Quizzes. Indeed, this manner of operation has quality control advantages. Each possible variant of a question can be reviewed before they are released to students, to ensure no degeneracy, special case or otherwise unwanted element of the question was accidentally introduced by the randomisation. In general there are between 5 and 20 variants of each question. The generation of these questions was facilitated by the use of a tool developed by one of the authors of this paper. The tool is described in the following section.

Alongside the Practice Quizzes are five summative computer assignments (denoted in the module as iCMAs: interactive computer marked assignments) which generally cover more than one study unit. These supplement the five TMAs of the module, and constitute 12% of the final module grade. The summative assignments aim to encourage students to make use of the Practice Quizzes: students are informed that the questions which appear in the iCMAs will be of a similar style to those of the Practice Quizzes, and encouraged to use the Practice Quizzes to familiarise themselves with the mathematical material, style of questions and the assessment system itself before taking a summative assessment.

The iCMAs also increase the number of points within the module at which a student must submit an assignment. This is aimed at helping students keep to a reasonable study timetable throughout the module. The iCMAs focus on small calculations, methods or the testing of definitions and concepts, thus allowing TMAs to concentrate on assessing problems requiring longer arguments, or skills that are less easy to assess automatically.

The feedback on summative iCMAs is delayed until after the assessment cut-off date, and students are only permitted one attempt at each. This model of operation was chosen for this module, but is not required by the VLE. Several other OU modules use a model closer to the Practice Quizzes where the student loses one third of the available marks for a question for each incorrect attempt. Questions within iCMAs are also randomised, and care was taken to ensure all the variants of each question are of equal difficulty. Randomising the questions in this way enables the same assessment questions to be used from one presentation to the next, saving staff costs in the long term.

The questions themselves, for both Practice Quizzes and iCMAs, were suggested by those preparing the corresponding study text of the module, then modified to fit the assessment system and implemented by the authors. During the development process the questions were subjected to extensive testing, both reviewing hard-copy of each of the possible question variants, and online testing of draft questions by a number of expert volunteers. The volunteers also tested for readability and accessibility across a range of computers and browsers.

In total, approximately 2500 questions were generated in support of this module. An example question showing the hint given after a second incorrect attempt and specific feedback to a common error is shown in Figure 1.

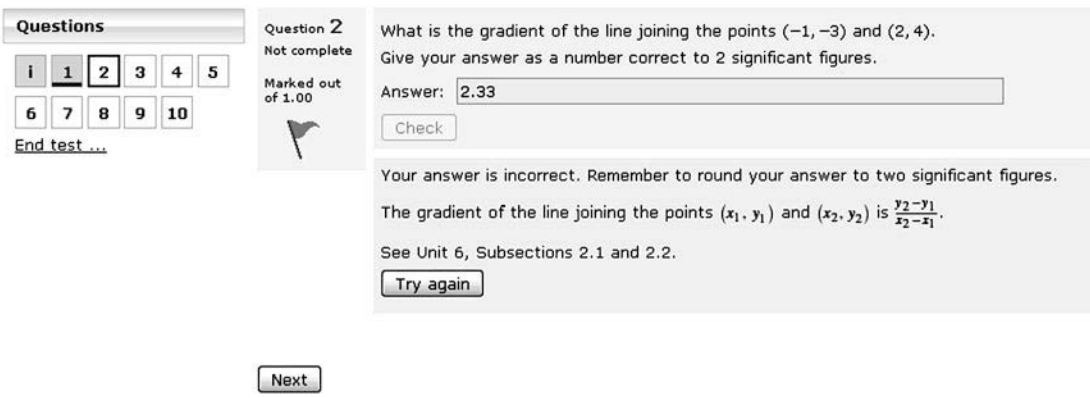

Figure 1: An example numerical question showing feedback to an incorrect response.

Links to access each Practice Quiz and iCMA are embedded in the Study Calendar on the module VLE site.

All the attempts at Practice Quizzes and summative iCMAs are recorded by the VLE and are available to a student's own tutor. In this way a tutor can, if they so wish, monitor a student's progress on the module and be proactive in offering advice and support.

2.3 iCMAtool

A program called iCMAtool was developed to generate variants of each question. The person writing each question uses an offline editor to produce a question template that consists mainly of standard Latex markup but also contains a code section that defines the variable parts of the template. (Here, Maple was used to process this section, but the other options are available.) The template is processed by iCMAtool to produce several outputs. One output is a Latex file of questions that enables offline reviewing of every question variant. Another output is a Moodle Quiz XML file [5] file for uploading to the VLE. A third output is a printable PDF booklet that can be sent to students without internet access.

An example Latex question template is shown in Figure 2.

```
\begin{numerical}
 \title{Simple multiplication}
\begin{code}
 a=op(N,[2,3,4,5,6])
 ans=2*a
\end{code}
\begin{questiontext}
What is $2\times \var{a}$?
\end{questiontext}
\begin{answer}[100]
\val{\var{ans}}
  \fb{$2\times \var{a}=\var{ans}$.}
\end{answer}
\end{numerical}
```

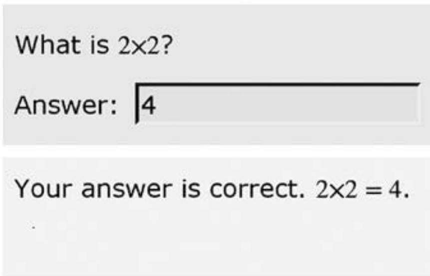

Figure 2: An example Latex question template, together with one of the questions generated.

3. Student usage and feedback

The marks obtained by students on each of the 5 summative assessments (iCMAs) and 14 practice assessments (PQs) are shown in Figure 3. Note that only scores where the student has explicitly finished the assessment by "submitting" their answers are included, and for each quiz only the best mark obtained by each student from their (possibly) multiple attempts has been included. It can be seen that the marks achieved are high: these assessment were viewed as trying to encourage and help the students with their study rather than discriminate among them, which was considered to be the function of the TMAs. As expected, the marks for the summative assignments decrease as the module progresses and the material becomes more difficult.

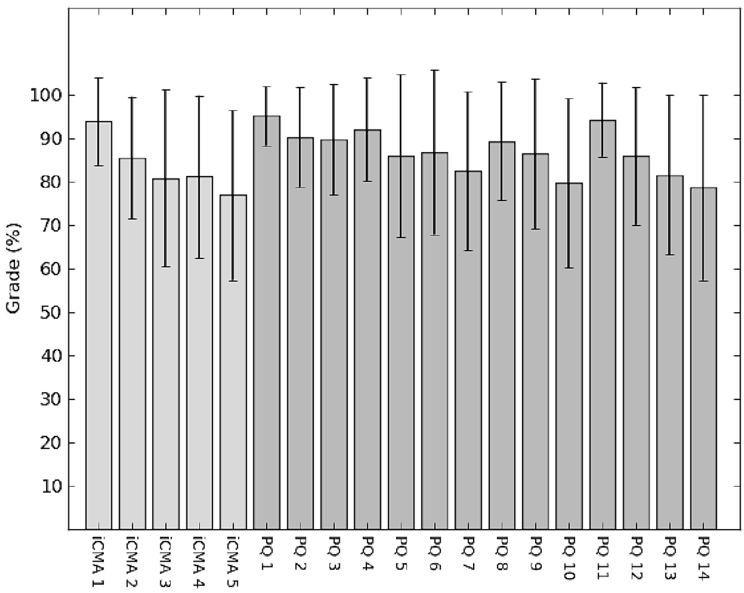

Figure 3: The mean mark obtained on each summative and practice assessment, and the standard deviation.

The number of attempts at each assessment is shown in Figure 4, which differentiates between the first and subsequent attempts of Practice Quizzes and those assessments which were not formally completed by "submitting" the assignment. The number of students registered at the start of this presentation of the module was 1561. As usual, the number of attempts at iCMAs decreases reflecting a retention rate that is typical for a module at this level with no entry requirements. For each summative iCMA there are also a number of students who did not "submit" their attempt, and hence the result would not be counted towards their final module result.

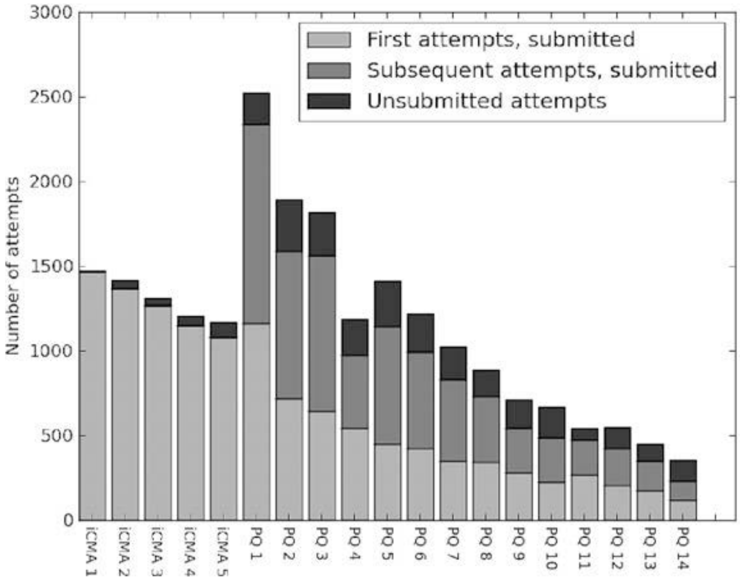

Figure 4: The number of attempts at each summative and practice assessment.

The number of individual students making use of the Practice Quizzes (as indicated by the number of first attempts) disappointingly decays rapidly over time. It is interesting to note that even at the start of the module, not all students attempted the first quiz. However, there were a large number of repeat attempts. The number of repeat attempts also generally decreases over time, although there is a marked increase on quiz 5, which corresponds to the point at which algebra is introduced.

Figure 5 charts the total number of students who made one or more attempts at each practice quiz. This also declines rapidly indicating that, perhaps, only 10 variants of each question are needed to satisfy the demand of students repeating the quiz. However, the chart also demonstrates that one student attempted one practice quiz 38 times.

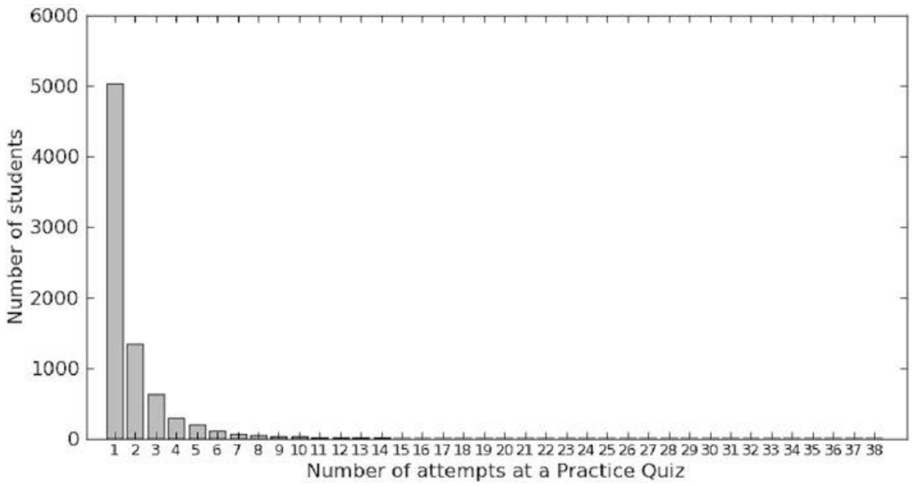

Figure 5: The number of students repeating attempts at Practice Quizzes.

Even though the quizzes were underutilised most of the comments made on the module online discussion forums were positive, for example:

> "The practice quizzes are a great way to build up knowledge."

"It's a great way to consolidate."

"I already submitted my iCMA ,I found that the practice quiz helped a lot."

"I like the practice quizzes, you can do lots of them until (hopefully) the lights come on."

"When I've finished a Unit I do the Practice Quiz over and over again until I get a really high score."

"I had a lot of trouble with this Unit too but found that if I kept doing the practice quizzes and when I got an answer wrong, working through the feedback on that question, it was a great help. I think I did the Practice Quizes about 8 times before I felt reasonably happy with It:)"

Some comments were more negative, reflecting the need to be online to attempt the quizzes and the low weighting given by the module assessment strategy:

"The practise quizzes are great - just wish I could do them when I'm sat under a tree in the forest waiting for my little girl to finish school."

"I don't enjoy the icma's at all and always do much better on the tma's. . . . I just think that they account for such a small percentage of your final mark that they really aren't worth worrying about."

4. Conclusions

Although the uptake of these assessments was less than hoped for, the student experience has been generally positive. The changes introduced in *Discovering Mathematics* have led to the module having a higher retention rate than its predecessor, although it is hard to attribute this to any specific change made.

There were a small number of technical issues relating to the online assessments that arose during this first presentation of the module. Some students were unable to view some of the font characters used within some questions, due to the use of old versions of web browsers. Some students also included superfluous text within answers to numerical questions (such as details of the rounding used), despite the quiz introduction and the individual questions giving instructions to the contrary. This led to some correct answers being marked as wrong. It is hoped this can be addressed with planned future improvements to the question types available. The range of question types currently available within the University also precludes the use of free-form algebraic text entry, but it is hope this will be addressed by the adoption of STACK [6].

In addition, there are certain groups of students who do not have internet access (for example, offender learners and submariners) hence who cannot access the quizzes. The ability to generate printed booklets from the Latex question source enables some support to be given to these groups, but alternatives are also under consideration.

The seeming success of using these assessments to help students learn is leading to their incorporation into further courses that are being developed.

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Acknowledgements

We would like to acknowledge the debt we owe to the rest of the *Discovering mathematics* module team and also to the external assessor for this module, Prof. Tony Croft. We should also like to thank those responsible of the development of the systems we used, and who gave us advice: Tim Hunt, Phil Butcher and Sally Jordan.

ASPs: Snakes or Ladders for Mathematics?

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Abstract

We review our experience at the University of Portsmouth over the past academic year, following a shift from home-grown to mass-produced electronic resources from an educational publisher, WileyPLUS. We present the results of a survey on the way mathematics students taking calculus units in their 1st and 2nd year viewed the shift towards WileyPLUS, and the opinions of staff involved in tutoring and supporting them. We discuss whether a personalised approach to teaching and learning can be maintained in a world of global education. Should the shift towards WileyPLUS become total, remain partial or simply be reversed? To what extent should we continue to integrate other resources, particularly assessment questions (e.g. from MapleTA, WebCT/Blackboard or PRS) with WileyPLUS?

Several other publishers of mathematics textbooks are providing comprehensive packages of interactive resources online. Application Services Providers (ASPs) create, store and deliver from their own server. Their resources include an electronic version of the textbook or e-book, supplementary materials, worked solutions, study guides, applets, formative assessment with extensive feedback and even summative assessment. Lecturers can customise these materials, most significantly by choosing questions for self-assessment tests and exams. Inevitably there are constraints on the extent to which they can modify or add to these materials.

Have ASPs improved the teaching and learning in our large classes of around 150 students? What about smaller classes? The stark choice between open source collaboration and commercial provision often polarises individual academics and even whole institutions. We argue that the ideal is a financially sound, hybrid model, which allows for greater customisation, sustainability, extension and interoperability than is currently available from either commercial providers or open source initiatives.

1. Introduction

Cleopatra, the last Pharoah of Ancient Egypt, allegedly killed herself by means of an asp bite on August 12th 30 BC. Over two thousand years later our question is whether the adoption of commercial teaching and learning software products from Applications Services Providers (ASPs) is a suicidal step. Academics cherish their independence of thought and individuality of their teaching, but are they being killed off by the emergence of integrated resources which seem to offer a complete on-line learning package?

In some ways the situation is not new. Lecturers have always had to make decisions about the adoption of standard mathematics textbooks, which chapters to include and how rigidly to follow the printed page. Some write their own in-house notes and ignore textbooks other than for reference or further reading; others develop supplementary materials such as their own notes to provide a digestible overview or worksheets to provide more problems and examples.

While the mathematics textbook has remained the mainstay of learning, some lecturers have invested major effort in developing their own e-teaching, e-learning or e-assessment resources to support their courses. Funded projects, both large and small, emerged in which academics collaborated on the production of electronic resources with or without associated paper resources. The sums of money spent on these projects have often been significant and the products variable in their impact and longevity. A major problem has always been the maintenance and ongoing development of resources, especially electronic resources, when the funding has dried up. The commercialisation of mathematical resources, which started up on public money, is unusual and the majority of "cottage industries" are doomed to long-term obsolescence. It is left to the goodwill and altruism of individual academics, who rarely have the time necessary for production and maintenance of high quality online resources.

Commercial publishers sometimes release new editions of mathematics textbooks every year and often every two or three years. Lecturers who adopt the textbooks are, in some sense, handing over control of their academic content and could even be regarded as committing intellectual suicide. Control can be maintained relatively easily by providing additional printed material to breathe life into a course. As commercial publishers develop increasingly sophisticated online resources to support or even replace textbooks, is the control of content being lost further? Do academics need to adapt their teaching again? ASPs create and store interactive resources for online delivery from their own server. Their commercial products are available from the publishers of several prominent mathematics textbooks: MyMathLab for Pearson (http://www.mymathlab.com), WebAssign for Thompson Learning/Cengage (http://www.webassign.net), MathPortal for W.H. Freeman and WileyPLUS for Wiley (http://wileyplus.com). Are ASPs to be regarded as a threat or an opportunity, snakes or ladders? We present our own views and those of our students based upon our initial experience of WileyPLUS.

2. Context: Staff and Students

The number of mathematics students at the University of Portsmouth increased dramatically between 2002 and 2010 with a greater than five-fold increase from under 30 to over 160 entering each year (Figure 1). In 2010/11 the total number of students exceeded 400 for the first time and the increase in total numbers is set to continue. During this period there were negligible increases in staff numbers and significant changes in staffing due to retirements. Mathematics often achieved top-4 National Student Survey and league table positions for both teaching and applied mathematics research. The entry grades of students also improved as the reputation of the department grew. We developed substantial use of Question Mark Perception for assessment during this period of growth in order to maintain the quality of teaching and learning for increasing numbers.

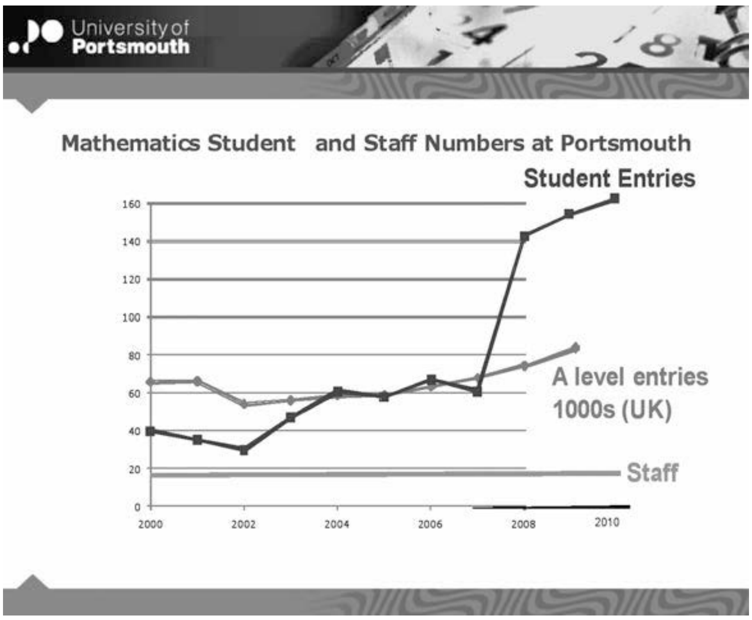

Figure 1: The Growth of Mathematics at the University of Portsmouth

3. Case Study: WileyPLUS

It is common for lecturers to have limited time to prepare printed resources for delivery of a new course unit/module. It is rare to have enough preparation time for the development of e-learning resources beyond a basic VLE "presence" by uploading notes and presentations. We describe a case study in which there was just two weeks to prepare for the delivery of a first year, second semester, 20-credit unit covering "further calculus and linear algebra" to over 150 students.

Existing resources included two recommended texts from different publishers, written lecture notes, the set of Question Mark Perception e-assessments (see above), in-class question banks for use with "clickers" (PRS handsets) and a minimal presence in the University "Victory" VLE. Weekly delivery was via two 1-hour lectures, one 2-hour "clicker" exercise classes, a 1-hour small group personal tutorial and a 1-hour assessment period during which seven tests were to be completed. Students were allowed to repeat tests to raise their marks, but credit could not be carried over and the same question, answered correctly at the first attempt, might well have to be repeated during subsequent attempts. Random parameters or algorithms were not available and the QM Perception questions developed over ten years earlier for formative and summative assessment were looking outdated and mathematically limited (Figure 2).

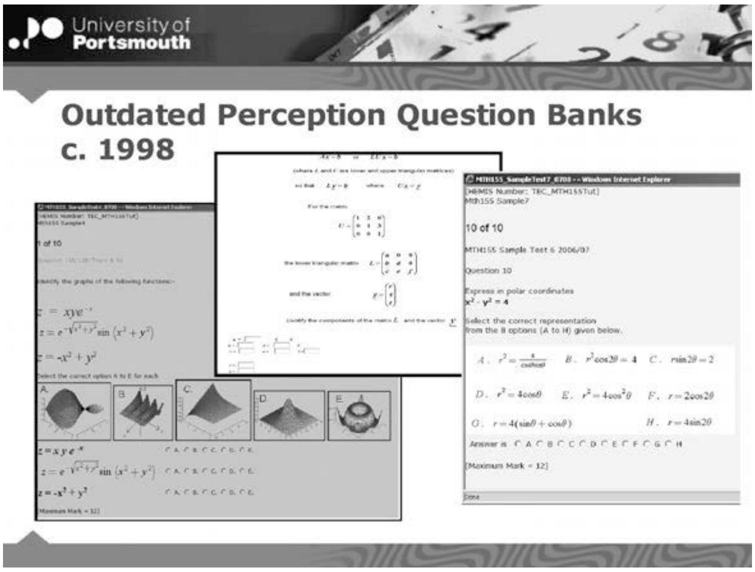

Figure 2: E-assessment Questions in Need of Replacement

With more time the questions could have been updated in QM Perception to eliminate some of the problems or converted to MapleTA to overcome the mathematical limitations. A provisional decision to introduce MyMathLab was switched to WileyPLUS for several reasons:

- it supported one of our adopted calculus textbooks [1]
- its underlying assessment engine was MapleTA (http://www.maplesoft.com/products/mapleta) which we already used
- it provided a comprehensive bank of online assessment questions including feedback and random-algorithms
- there was local Wiley support provided at short notice to set up WileyPLUS access for road testing and live delivery, to deal with technical, especially assessment, issues that arose and to introduce the product to both tutorial staff and students.

The old QM Perception e-assessment provided our "safety net" in the event of problems and the decision was taken to adopt WileyPLUS [2] immediately as a "live pilot". For the trial semester, WileyPLUS was provided free-of-charge, since students had already purchased the associated textbook. There were many advantages of using WileyPLUS:

- a complete electronic copy of the text was available to all student regardless of whether they had purchased a hard copy
- large online question banks with the underlying MapleTA/MapleNet engine
- graded feedback with links from assessments to hints, solutions, tutorials and the book itself.
- learning design underpinning its overall structure
- presentations for lectures including PRS "clicker" questions and summaries
- some applets for interactive activities
- instructor guidance and resources
- extra question banks for use within our "Victory" WebCT VLE (Figure 3)

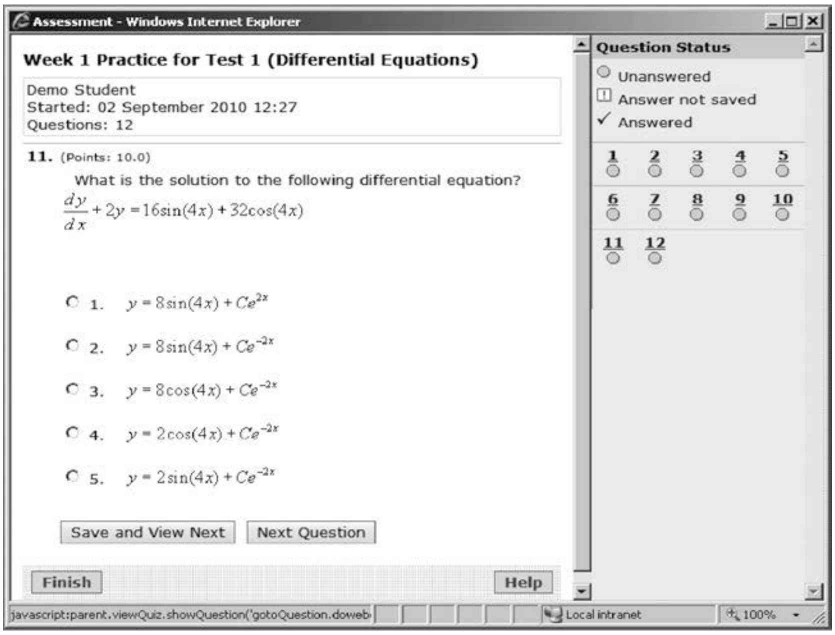

Figure 3: E-Assessment Questions Exported from WileyPLUS into WebCT

The graded feedback (Figure 4) is a particularly strong feature, since it allows progressively more detailed help to be given after each unsuccessful attempt at a question.

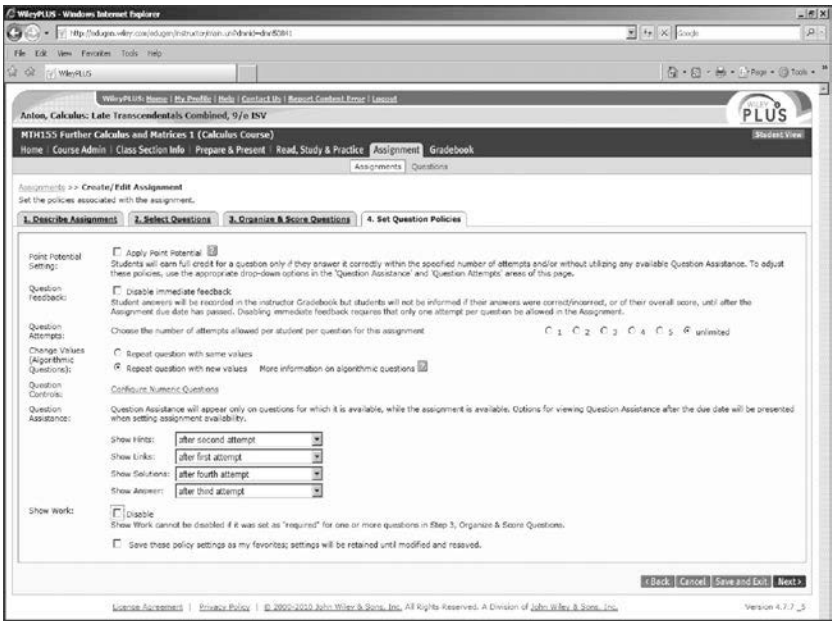

Figure 4: Graded Feedback Options in WileyPLUS

There were also disadvantages in using WileyPLUS (Version 4.7.8) :

- the current product is designed for learning not exams, with an emphasis on formative, not summative assessment
- textbook-based lecture slides which look formulaic and uninteresting
- major limitations on question customisation and in-house authoring
- an inability to import existing MapleTA or QML questions
- an inability to correct errors or modify content
- an inability to hide/show content or perform basic VLE operations to modify the display, e.g. for accessibility

When there were mistakes in e-assessment questions, it was only possible to report the error and avoid using them. The response time for having such corrections made was too long.

Both WileyPLUS and local resources, including MapleTA assessments, were linked through the VLE, which acted as a one-stop shop for the course unit. Formative WileyPLUS assessments (http://wileyplus.com) included:

- standard practice tests for all sub-topics covered in the text
- custom practice tests generated by selecting questions from the bank and setting up appropriate delivery and feedback options
- question selection by difficulty, learning objective or type
- varied answer input, including interactive graphs
- unlimited attempts on algorithmic-randomised questions

Formative MapleTA assessments were authored in-house and are available free of charge for users of MapleTA at http://userweb.port.ac.uk/~mccabeem/mapleta. MapleTA does not include some WileyPLUS features, such as interactive graphical questions (Figure 5) and a smart symbolic input palette (Figures 6 and 7), but this drawback can be weighed against the benefits of greater flexibility and control over question authoring, modification and delivery.

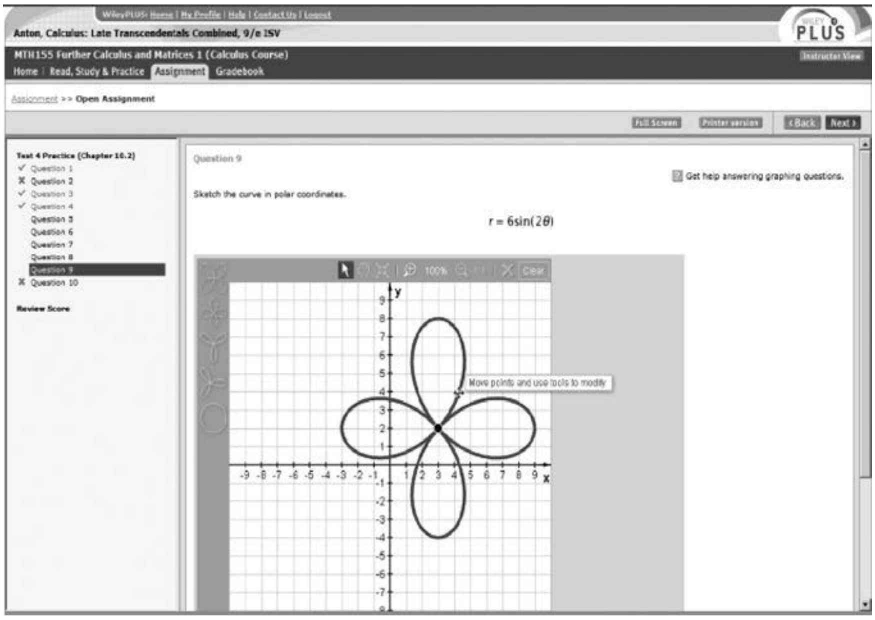

Figure 5: Graphical Input in WileyPLUS

Since a large bank of MCQ and numeric formative assessment questions were available to students within WebCT, they were required to deal with three different interfaces for answering questions. The lack of interoperability

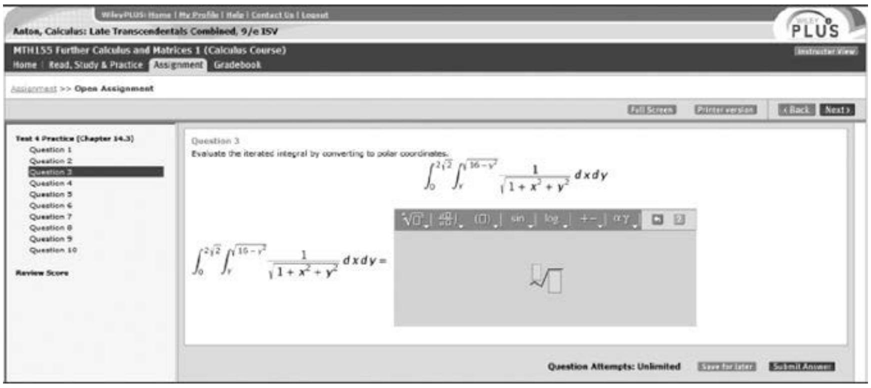

Figure 6: Symbolic Input in WileyPLUS using a Palette

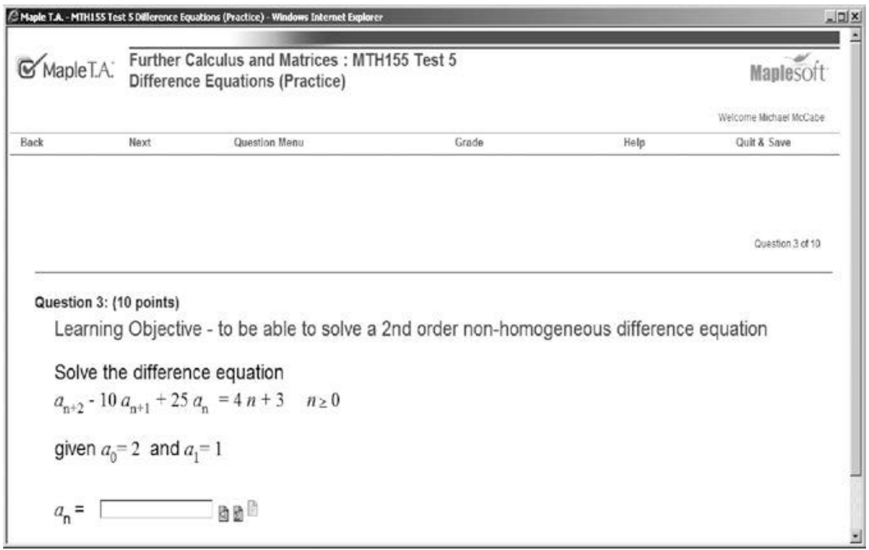

Figure 7: Symbolic Input in MapleTA

between WileyPLUS, MapleTA and WebCT prevented any integration of question banks. Given that MapleTA underpins WileyPLUS, that limitation might be overcome in the future, but remains an issue for the present.

4. Summative Assessment

Of seven exam tests taken by students for summative assessment, four were in WileyPLUS and three in MapleTA. This mix of delivery allowed us to compare their experience in using each of them.

For WileyPLUS:

- custom exam tests were created by selection of suitable questions from the bank
- "baseball" questions were set up so that up to three attempts at each question could be made without any penalty
- each test could be re-entered by students during one or more exam sessions to raise their mark
- questions were selected with a balance of difficulties, learning objectives and types
- administratively time-consuming workarounds had to be developed to account for the lack of test passwords in the WileyPLUS system

The cumulative scoring of "baseball" questions was designed to promote student confidence by allowing them to keep marks from all successful answers without having to repeat similar questions. Traditionally a student would have been expected to repeat a "knockout" test in its entirety with any score below the required threshold being effectively the same as no attempt at all.

Little use was made of the limited question authoring available in WileyPLUS, since it is limited to basic question types (MCQ, text, numeric, essay) using plain text only.

For MapleTA:

- exam security was greatly strengthened through the use of passwords and the administrative overload was reduced
- the setting up of tests with randomised questions was more flexible and easier to control
- all tests were "knockout" in the sense that each one had to be retaken if it was not passed, although it would have been possible to set up similar "baseball" questions to WileyPLUS by allowing up to three attempts

Many issues arise in using the resources in WileyPLUS and, to a lesser extent, MapleTA :

- The products are subject to future development and the release of new versions. There is no control over long-term continuity and stability of existing assessments in the future. Upward compatibility may not be possible and further time may need to be spent in setting up existing tests for annual use.
- The non-uniformity and comparability of questions in terms of their length, time required and scoring
- A need for discrimination between formative questions suitable for learning and summative questions suitable for examination.
- Partial credit for answers which are incomplete or inaccurate

5. Evaluation Method

We were keen to get rapid feedback from students who were learning from on-line resources, introduced on an extremely short timescale. Since it was a "live pilot" we initially relied on verbal comments, which reassured us that students were benefiting from their WileyPLUS experience and that we did not need to revert to existing resources. The "baseball" questions allowed us to track student progress and results were good.

A simple questionnaire was distributed which sought views on all their e-learning resources, with a particular focus on WileyPLUS. For each resource they were asked to rate their features on a Likert scale, e.g. 1 = essential to learning 6 = no use at all. A further set of open-ended question sought positive and negative responses. 68 out of 150 "Further Calculus and Matrices" students at level 1, who had used WileyPLUS throughout, responded. A smaller set of responses were also obtained from 11 out of 120 "Calculus of Several Variables" students at level 2, who had only used WileyPLUS in their final weeks of study in preparation for a final exam.

Evaluation of VLE ('Victory', in WebCT)

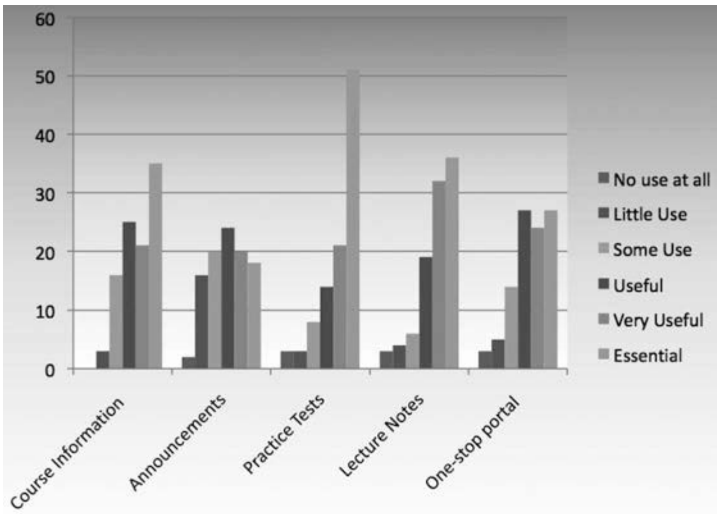

Figure 8: Usefulness of Victory Resources for Learning (%ages)

The Likert-scale questions showed that practice tests were valued most (50% essential for learning), 'just-in-time' lecture notes were highly valued, and the Victory (WebCT) site was also valued for providing a one-stop portal and course information. Victory was valued for providing a one-stop portal (27%) and course information (35%).

The open-ended questions invited student responses on any aspect of the VLE, which is used for most course units. The number of responses on a given point are shown in brackets. There were two overwhelming positives:

1. The VLE provided a necessary one-stop shop linking access to all resources, including WileyPLUS and MapleTA. It was the glue which held all the components together. (12)
2. 'Just-in-time' handwritten lecture notes (Figure 8) provided on a weekly basis as PDF files were greatly appreciated. (16) This came as a surprise given that students already had full access to WileyPLUS and that the rough notes were simply scanned after lectures. Students may have valued them for their focus on process rather than product, for demonstrating mathematical thinking in a digestible summary, for offering a more personalised approach which could be linked to what was said in lectures or for the local course information included. The responses did not go into more details.

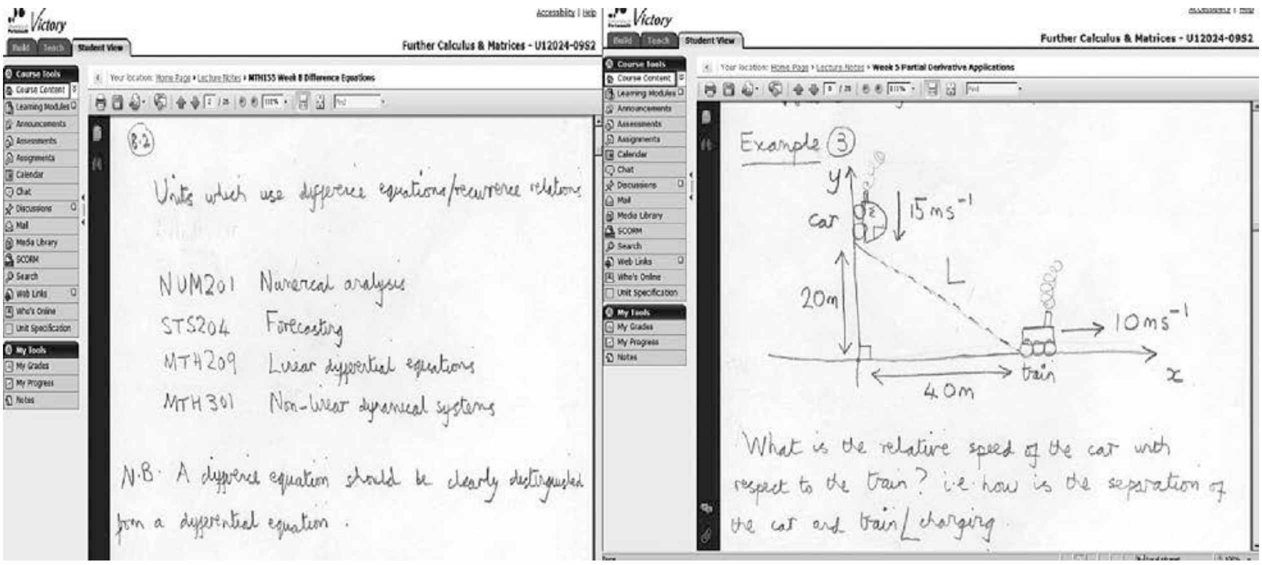

Figure 9: "Just-In-Time" Handwritten Lecture Notes

Amongst the negatives, unreliability (7) , GUI navigation issues (7) and mathematical limitations (5) featured more predictably. None commented on the extra e-assessment resources available within the VLE, suggesting that their lack of integration with WileyPLUS led to them being largely ignored.

6. Evaluation of MapleTA

We were interested in comparing student views on our 'in-house' MapleTA questions as opposed to the "outsourced" WileyPLUS questions. Many commented positively on the value of having a large number of MapleTA practice questions/tests (13) and the ease-of-use (12). The main negative comments were about difficulties with the syntax or format of input, arising from the lack of the palette tool in WileyPLUS (10), unfair or inflexible marking and lack of partial credit (5) and limited feedback such as hints, links and full solutions, which were more regularly available in WileyPLUS (6). A small number also identified the fact that tests were of the "knockout" variety requiring all questions to be repeated rather than having "baseball" questions allowing them to have three attempts (4).

7. Evaluation of WileyPLUS

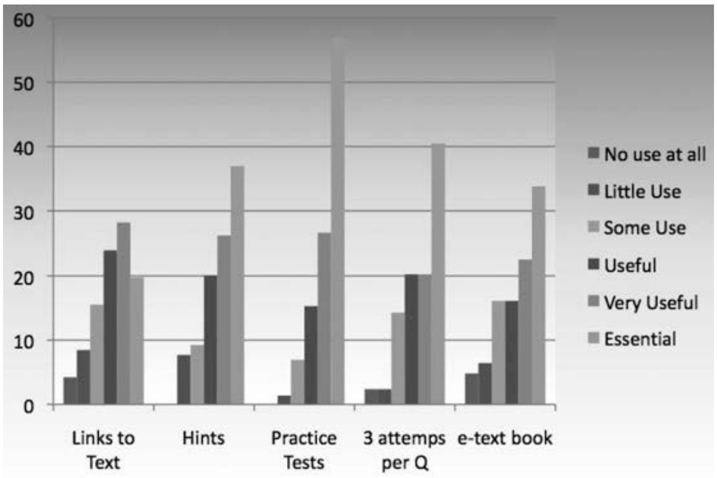

Figure 10: Usefulness of WileyPLUS Resources for Learning (%ages)

The Likert-scale responses indicated very high value for the practice tests (60% essential for learning), followed by the 3 attempts per questions, and the hints and e-textbook. The figures for MapleTA also indicated very high value for the MapleTA practice tests (the other questions did not apply to MapleTA).

Amongst all the open-ended responses, it was the availability of "baseball" questions and cumulative scoring in WileyPLUS that provoked the greatest positive comment (17). The ability to have three attempts at different instances of the same question without penalty seemed to offer greater encouragement to students and a more constructive approach to learning. Other positive comments were on the extensive feedback through book links, hints, solutions and tutorials (18) and the numerous/varied practice set of questions (9). Negative comments related to input difficulties despite the use of symbol palettes (4), GUI issues, e.g. need for multiple windows, and unfair marking (3). One student made the interesting negative comment that it was not always possible to repeat a question when a given instance had been answered correctly. It had incorrectly been assumed that they would not wish to continue practicing on similar questions to reinforce their understanding. Amongst other one-off comments was a reference to "friendly, calm and helpful invigilators" who support the delivery of e-assessments. Such a simple observation is easily overlooked when the focus of an assessment is on its technical delivery and content.

8. Future Directions in a Commercial World

Our experience in using WileyPLUS for calculus has been sufficiently positive that its use has already been extended to linear algebra [3] [4] in 2010/11. It is starting to provide some of the e-learning tools and the content necessary for undergraduate mathematics. At the University of Portsmouth the curriculum for every course, including mathematics, is undergoing a complete revision for 2012/13. Our present expectations are that WileyPLUS will continue to be adopted and integrated into our courses as part of that fresh start. This may well coincide with the next major release of WileyPLUS 5.0, which it is hoped will incorporate such features as exam security with password protection and a greater scope for question authoring, customisation and interoperability. The University also will need to change its VLE from WebCT in 2013, and ongoing change can be expected in the way that e-learning is required to be delivered. The question is whether it is possible to maintain "in-house" development of our own e-learning resources or whether we should become increasingly reliant upon those developed by external providers, either commercial or non-commercial.

The stark choice between open source collaboration and commercial provision often polarises individual academics and even whole institutions, but maybe there can be greater cooperation between commercial and non-commercial developers. It may be that universities themselves become increasingly commercialised, but few other than perhaps the Open University are likely to have the infrastructure necessary to produce the equivalent to WileyPLUS.

We argue that the ideal is a financially sound, hybrid model, which allows for greater customisation, sustainability, extension and interoperability than is currently available from either commercial providers or open source initiatives. There has always been some degree of hybridisation between commercial and non-commercial tools and resources. Commercial software has often been used to develop e-learning and e-assessment resources for non-commercial distribution within HE. Individuals or small groups have developed e-assessment question banks which are freely distributed, while the underlying software remains commercial. Some commercial publishers, e.g. Bedford, Freeman and Worth, develop e-packs for VLEs which can then be freely modified and extended. Conversely, non-commercial software can be used as the basis for commercial products, e.g. Red Hat (http://www.redhat.com/products).

	free resources and add-ons	commercial resources and add-ons
free tool/package	fully open source **STACK, OpenMark, Moodle**	hybrid **Red Hat**
commercial tool/package	hybrid **MapleTA banks (M^3)** **Perception (MG)** **WebCT e-packs** **Toolbook (Mathwise)**	fully outsourced commercial **WileyPLUS** **MyMathLab** **WebAssign** **CALMAT**

Table 1: Commercial vs. Non-Commercial in E-learning and E-assessment

Idealists will always argue for completely free and open software development in the academic world, but this is likely to remain an ideal. Yet WileyPLUS is in danger of becoming as inflexible as a textbook, if it limits customisation, modification and extension of its commercial resources. Furthermore any such changes need to be sustainable when new versions of the product become available. The situation is similar to the release of a new edition of a textbook, requiring a lecturer to update notes and supporting material, but only worse. Written notes may only require a revision to a list of recommended exercises or chapter numbers. Significant effort may be needed to revise e-assessments in WileyPLUS.

At the University of Portsmouth the main driver of change has been a sharp rise in student numbers, the demand for modern e-learning resources and need for both formative and summative e-assessment. It makes sound economic sense to buy in affordable commercial products for large first and second year units. We were able to introduce the calculus course to students and tutorial staff on an extremely short timescale with low overhead. Weekly small group personal tutorials were supported by WileyPLUS both through printed questions and through direct use of the e-assessments.

Ideally the staff time freed up can become available for tailoring the resources to local need or doing other things. A problem with WileyPLUS is the standalone nature of each product in supporting an existing textbook. The printed word remains a constraining factor and it is impossible for a lecturer to mix-and-match e-learning resources or assessments from separate WileyPLUS courses. For standard courses such as first year calculus that may be fine, but elsewhere in the curriculum it is likely to be highly restrictive. We have already noted that there is no distinction in WileyPLUS between those questions which are appropriate for learning, i.e. formative, and those questions which are appropriate for examination, i.e. summative. Our experience suggests that clearer guidance on question use would be appropriate when (and if) the product incorporates greater exam security. Identifying questions by their difficulty, question type or learning objective is insufficient.

To sum up, there are two distinct challenges that face universities – in assessment, and in teaching and learning.

In assessment, there is a need for customisation, which includes the provision of:

- Full authoring tools for academics to create additional assessment materials.

- Security access, and administrative processes and rights, for managing summative assessment, e.g. timed passwords for assessments.
- Interoperability, for instance between MapleTA-based provider materials and user-created materials.

These are essentially technical and systems issues and they need to be addressed rapidly by any provider wishing to maintain market share. They require minimal hybridisation and collaboration between private and public organisations, and would provide a possible modus operandi for collaboration and integration across university and commercial providers.

The challenges for teaching and learning, however, are much broader, and include the need for:

- Resources to be designed primarily for learning, and therefore assessment needs to be designed primarily as formative benchmarking, rather than summative assessment. (This is, as it happens, already the case in WileyPLUS).
- Summative assessment is also required, but it needs to be integrated as far as possible with formative benchmarking. Portsmouth has developed over several years what is in fact a hybrid between summative and formative assessment, and the use of 'three attempts per question' that we have implemented in WileyPLUS is an excellent extension of that principle.
- The tracking data base, which captures information on the use and progress of learners, needs to be designed to provide reports primarily for teaching and for learning, rather than primarily for administration and for management evaluation (necessary as these are).

ASPs such as WileyPLUS provide quite a lot of useful data, which can, already, be exported to administrative reports (which is good), but very little in the way of reports that are of immediate use to learners, tutors and lecturers, even though much of the data that could be used to generate these reports is already being captured.

If ASPs are to provide an integrated 'service', the real challenge in an age of web2.0 and 3.0 and social software is to provide not only the integration of teaching, learning, benchmarking and assessment, but also to provide wider integration with, and links to, the growing wealth of resources and interactive communities available through the Internet.

It is a paradox that people want total flexibility to customise a product such as WileyPLUS in a myriad of different ways, but in practice few have the time or inclination to make those changes. WileyPLUS clearly has far to go in its development, and it is important that commercial developers are aware of academic needs. Some may regard the adoption of ASPs as intellectual suicide on par with Cleopatra, which limits the independence and freedom of academics. In the future, we would like to maintain the benefits which have arisen from our use of WileyPLUS, while eliminating the drawbacks that we have identified from our own experience and student feedback. ASPs are unlikely to become extinct, but they may need to adapt to survive.

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DCU Voluntary Maths Tuition Programme

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Abstract

The Dublin City University (DCU) Voluntary Mathematics Tuition Programme was established in September 2009 as a joint initiative between DCU Mathematics Learning Centre, DCU Access Service and Ballymun Initiative for Third-Level Education. The programme aims to increase the confidence levels and mathematical standards of local secondary school pupils from disadvantaged areas, while raising the profile of mathematics within the schools, through an intervention in which DCU students provide free, one-to-one mathematics tuition on a weekly basis. The requirement of Leaving Certificate Ordinary Level mathematics for entry into third-level in Ireland had been found to be a barrier to these pupils, preventing progression into higher education, and this motivated the development of the programme, which was modelled on similar schemes operating in a number of other universities.

Exceeding initial expectations, a total of ninety DCU students volunteered to be part of the programme, and more than seventy pupils from a local school were involved. The feedback from both tutors and pupils was extremely positive, with both groups identifying significant personal benefits as a result, as well as the mathematics results of school pupils increasing overall. In addition, the school principal singled out the programme as having made third-level education seem like "a reasonable expectation for students from our school".

Here, we discuss the practicalities surrounding the set-up and implementation of such a scheme, including the workload involved, challenges faced and specific comments received from tutors and pupils. In particular, we consider what can be achieved in outreach programmes for which there is little or no funding available. In the coming years, facing increasing cuts in education budgets, the successful implementation of schemes such as this one will become ever more important.

1. Introduction

In the Irish education system, students take two national examinations during the course of their second-level education: the Junior Certificate, at the end of their third year in secondary school, and the Leaving Certificate, at the end of their sixth year [1]. For the latter, students take at least six subjects, with most taking seven or eight, as their best six results are counted for entry into third-level programmes. The Leaving Certificate Mathematics examination can be taken at three different levels: Foundation, Ordinary and Higher. In order to pass a subject, a student must obtain 40% or higher. Often, to qualify for a third-level degree programme, at least a pass in Ordinary Level mathematics is required, with Foundation Level mathematics not normally accepted, although some exceptions are made. Students must also obtain sufficient "points" from their other subjects and satisfy various other criteria, but the specifics of this need not concern us for the purposes of this paper. The requirement of Leaving Certificate Ordinary Level mathematics for entry into third-level had been found to be a barrier to some pupils, preventing progression into higher education [2].

Dublin City University (DCU) is situated to the north of Dublin city, and one of its local neighbourhoods is a socio-economically deprived area known as Ballymun. In 1990, Ballymun Initiative for Third-Level Education (BITE) was established with the aim of encouraging students from the area to fully engage with education, and, to date, BITE has successfully supported over 170 students through third-level education [3]. As a result of detailed discussions between BITE, DCU Access Service (which works with students from socio-economically deprived backgrounds) and DCU Mathematics Learning Centre, along with advice shared by colleagues in the University of Limerick who had previously organised a similar programme [4], DCU Voluntary Mathematics Tuition Programme was formally established in September 2009. The aims of the programme are to increase the confidence levels and mathematical standards of local secondary school pupils, while raising the profile of mathematics within the schools, through an intervention in which DCU students provide free, one-to-one mathematics tuition on a weekly basis.

2. Implementation of the Programme

Clearly, the success of the programme depended entirely on the levels of involvement of both the DCU students and the school pupils. Requests for volunteers from DCU were made in the form of posters, announcements during Orientation Week, and emails sent to class lists by the Mathematics Learning Centre. As a result, a total of 90 DCU students from a range of backgrounds and disciplines volunteered to be tutors in the Voluntary Mathematics Tuition Programme.

After a brief introductory meeting in which some basic training and guidelines regarding working with young people were covered, the tutors attended the school once a week for an hour and helped pupils with homework, revision and exam preparation. The pupils involved were from third, fifth and sixth years and studying a range of Higher, Ordinary and Foundation Level mathematics, with up to seventy pupils participating in the programme. Like many courses in DCU, the comprehensive school involved in the programme does not schedule classes on Wednesday afternoons, so this was chosen as an ideal time for tuition. The voluntary tuition sessions took place from 14th October – 16th December and 10th February – 28th April.

A closing ceremony was held for tutors and pupils on 21st April, in which certificates were distributed and pupils received "goodie packs" provided by DCU Access Service. Martin Conry, Secretary of DCU, who attended the ceremony, remarked that it was "inspiring to see DCU students volunteer so much of their time to help those in the communities surrounding DCU".

3. Challenges Faced

Within any programme of this kind, there are numerous challenges to be faced, and many of these will depend upon the particular school involved. Given that there is a full-time BITE representative working within the comprehensive school involved in this programme, she took responsibility for pairing pupils with suitable tutors, and contacted the tutors via email on a weekly basis to let them know if their pupil would attend tuition. Despite these efforts, absenteeism was a frequent problem, particularly in relation to the school pupils, often due to poor attendance at school in general. In addition, in order to maximise the tuition received by each pupil, it was decided that pupils should come every week, even if it was known in advance that their tutor was unable to attend on occasion. On such days, they were assigned a different tutor. As a result, pupils and tutors did not always work in the same pairings, which was not ideal. Another challenge was that the pupils' timekeeping was often poor, with some arriving up to fifteen minutes late for their hour-long session, and many times they did not come equipped with mathematics textbooks, past examination papers, calculators or sometimes even pens and paper, despite having been at school all morning.

4. Impact of the Programme

During the course of the year, sixth-year pupils took six assessments as part of their mathematics classes. Results were available for 36 pupils who took part in the programme. 15 of the pupils took Ordinary Level for their Leaving Certificate and the average increase in their results was 41%. The nine pupils who were taking Foundation Level from the start of the year saw an average increase of 63% in their results, with one pupil recording an improvement of 103%. 12 students who began the year studying Ordinary Level have seen their results drop by an average of 49% and as a result will take Foundation Level. This decrease can in a large part be explained by poor school attendance in both fifth and sixth year, and emphasises the need for intervention in younger years.

However, in addition to an academic impact, the programme was also successful in promoting third level education within the school. The principal, Pat O'Dowd, commented that

> *"Not only did maths grades increase overall, but meeting a third level student, often for the first time, every week has helped break down the perceived barriers attached to higher education for students from the school. Having DCU on the school's doorstep did not seem to have any great relevance to our students until this initiative. As a result of the programme not only has the profile of maths been raised within the school, but more importantly…DCU is now seen as an accessible resource in Ballymun and third level education has been promoted as a reasonable expectation for students from our school."*

5. Feedback from Pupils and Tutors

In order to improve the programme in coming years, pupils and tutors who took part were asked to complete anonymous surveys to obtain their feedback about the initiative. Twenty-seven responses were received from school pupils, about 80% of which were from sixth-year pupils, with the remainder from third-years. When asked what they found helpful about the programme, pupils most commonly referred to the fact that they received one-to-one help; that the tutors were patient and helpful and "teach at your level"; and that the tuition took place in their school. In addition, the vast majority felt that their maths grades had improved. In response to how the programme had changed how they feel about their maths, most indicated their increased confidence, whereas "before it was very hard and stressful", and the fact that they felt better prepared for examinations, as tutors "showed me how to break questions down." In relation to improvements that could be made to the programme, many stressed the importance of working with the same tutor every week, and several asked for longer or more sessions per week. When asked what they liked best about the programme, the overwhelming majority commented on the tutors, with the importance of the relationship built up over the year again coming to the forefront: "My tutor was there everyday for me." The other factor that pupils identified was simply the opportunity to learn mathematics: "Been able to finally understanding maths work." Overall, their responses are perhaps best summarised in two short comments: "The tutors were very nice and treated us with respect and it was a great experience" and "I learned new things and made a new friend for life."

For the tutor survey, a total of 40 responses were received. All those who responded found taking part in the programme to be a positive experience for themselves, citing benefits such as "interacting with younger people" and "giving back to the community". Some enjoyed sharing their own love of maths, saying they were "happy to show them that maths can be an interesting and manageable subject"; two tutors even said that they now "think becoming a maths teacher might be the way for me to go". Several referred to the fact that "seeing how much they appreciated our effort made the experience all the more worthwhile" and felt that they learned a lot about themselves in the process of the tuition.

When asked to describe the impact they saw the programme having on the pupils they tutored, most referred to the increase they saw in their pupils' self-confidence – "I could see that he was proud of himself and encouraged when he was able to do the maths that I had showed him" – as well as marked improvements in their mathematics – "it was good particularly when your student goes from not being able to add 2 and 2 to being able to do a quadratic". They spoke of pupils "engaging more with actually trying to understand what they were doing when solving problems, rather than just memorising solving procedures" and described how the pupils "had piles of potential" and "wanted to be there to learn maths and they tried there (sic) best." They also referred to the relationships they forged with pupils over the course of the year – "The student I tutored became very attached, she kind of relied on me to help her learn…" – and that they spoke to pupils about the fact that there are "so many options with maths and that they shouldn't just consider it a subject that they need to get through for their exams and forget about it." Several tutors described the process of turning around pupils' poor self-belief in their mathematical abilities: "The first thing every student told me at the start was that they were rubbish at maths, but when we started going through the questions and giving them pointers I was able to show them how much they knew, and changed their attitude towards the subject" and "Well, one girl i helped was very negative towards maths for some time, and i just took my time and listened to her. And then explained the maths in simple form, giving her some useful tips to remember things. And she seemed to understand it far better by the end of the session and seemed delighted with herself that she could do it." In terms of challenges they encountered, the most common was in relation to the teaching of mathematics itself, with tutors observing that "at first it was hard to explain things that I already knew" and "explaining maths in simple English was difficult sometimes". Some tutors mentioned that their pupils did not always turn up for tuition, meaning that they had a wasted journey or worked with a different pupil that day. Others alluded to the difficulty of getting some pupils to focus on their work, or have the correct materials with them.

In common with the pupils, when asked for improvements that could be made to the programme, tutors emphasised the importance of working with the same pupil every week, in order to effect real improvements in their mathematics, although they noted the difficulty of this due to absenteeism. They also suggested closer liaisons with the teachers and more structured work to do during sessions. Several tutors also requested more specific pointers on teaching mathematics, so this will be incorporated into the training session for next year's tutors. Overall, tutors were very positive about the programme – "I am proud to say that I was a part of it" – and keen to take part in the coming year.

6. Conclusion

Overall, the Voluntary Mathematics Tuition Programme has exceeded our expectations, due in no small part to the excellent work of both the DCU students and the school pupils involved in the programme. Both groups seem to have benefited greatly from the experience. However, despite the intervention, no pupil from the comprehensive school took Higher Level mathematics for their Leaving Certificate in June. Therefore, while continuing to run the programme within the school, we hope to extend it to provide additional support to pupils from transition, fifth or sixth year who are considering taking Higher Level mathematics, with these pupils attending one-to-one tuition within DCU twice a week. In addition, subject to volunteer tutor numbers, we hope to encourage more pupils from younger years to take part in the initiative.

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Stimulating Techniques in Entry-level Mathematics

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Abstract

This paper reports the outcomes of a project which aimed to build on existing best practice in diagnostic testing of mathematics. The project took existing diagnostic resources and developed tests for the STACK computer aided assessment system. This paper reports on the test design, implementation and how the results of the test were analysed and reported to students and staff.

1. Introduction

In April 2010 the National HE STEM Programme funded a mini-project with the title *Stimulating Techniques in Entry-level Mathematics (STEM) with the STACK computer aided assessment (CAA) system*. The aim of this project was to take existing "diagnostic tests" in core mathematics and develop similar automatic tests for the STACK computer aided assessment system. This paper reports on the outcomes of this project.

1.1 The STACK CAA system

STACK, a System for Teaching and Assessment using a Computer algebra Kernel, is a computer aided assessment system for mathematics. A student enters their answer as a mathematical expression, whose mathematical properties are then established with the use of the computer algebra system Maxima. For many questions the teacher will seek to establish that the answer is (i) algebraically *equivalent* to the correct answer and (ii) in the appropriate *form*, (e.g. factored). However, the answer need not be unique. STACK uses the computer algebra system Maxima to

- randomly generate problems in a structured mathematical way;
- establish the mathematical properties of expressions entered by the student;
- generate outcomes in the form of a numerical score; feedback which may include mathematical computations of the student's answer; and a *note* for later statistical analysis.

STACK was designed and developed at the University of Birmingham, United Kingdom, with support from the Higher Education Academy's Maths, Stats and OR Network. More details of STACK can be found in, for example [1], [2], [3], and [4].

1.2 Diagnostic testing and the nature of effective feedback

Diagnostic testing in mathematics is widespread and has been used for many years. For example [5] found

1. At least 60 Departments of Mathematics, Physics and Engineering give diagnostic tests in mathematics to their new undergraduates. [...]

5. Diagnostic tests play an important part in
   - identifying students at risk of failing because of their mathematical deficiencies,
   - targeting remedial help,
   - designing programmes and modules that take account of general levels of mathematical attainments, and
   - removing unrealistic staff expectations.

Indeed, [5] goes on to recommend that "*students embarking on mathematics-based degree courses should have a diagnostic test on entry*" and that "*prompt and effective support should be available to students whose mathematical background is found wanting by the tests*". An extensive series of case studies in diagnostic testing was published as [6]. Furthermore, there have been specific CAA systems for diagnostic testing such as DIAGNOSYS, [7]. However, there is currently no such diagnostic test for the STACK CAA system.

Notice the importance of feedback, both to students and staff. It is almost axiomatic that "feedback promotes learning". However, the research evidence on this matter appears less clear. For example, the meta-analysis of Kluger, see [8], examined about 3000 educational studies and found that over one third of formative feedback interventions *decreased performance*: a counterintuitive and largely ignored outcome. They concluded that it is not feedback itself but the nature of the feedback which determines whether it is effective. In particular, feedback which concentrates on specific *task related* features and on *how to improve* is found to be effective in a formative setting, whereas feedback which focuses on the *self* is detrimental. A low end-of-test summary mark - hardly a specific form of feedback - may be interpreted as a personal and general comment on the ability of the student, whereas detailed feedback on each task points to where improvement can be made. Following on from this work in [9] Shute recommends that we should be *"cautious about providing overall grades"*, and *"use `praise' sparingly, if at all"*. Furthermore, she suggests we *"avoid using progressive hints that always terminate with the correct answer."* and *"minimize the use of extensive error analyses and diagnosis"*. Clearly it is necessary to gain some broad sense of what a student can and cannot do in order to target help so that they may improve their performance on the task. However, for the purposes of this project we fall short of the level of detail used in the DIAGNOSYS system, see [7], and other intelligent tutoring systems, see for example [10].

Hence we decided that numerical scores on individual questions and tests will *not* be available to students. Instead, text based feedback (which includes e.g. "correct answer") will be used. Further, diagnosis will be at broad topic level, e.g. "algebra", or "calculus", not at the minute detail of individual technical skills.

We also decided to implement confidence based testing. Each question contains a "slider" to enable the student to indicate their confidence with each answer, see [11].

2. Core skills in mathematics

The majority of students taking a diagnostic test such as ours will have been educated in the United Kingdom school system. The purpose of these tests is to assess skills at the start of a university course. We therefore have a responsibility to include within the assessment only those topics which students can be expected to have been taught, and to assess them in a manner similar to the examination questions with which they are familiar. If the users of these tests are not happy with the skills of incoming students then it is the responsibility of the university department to educate their students in different ways. Therefore we took the Qualifications and Curriculum Agency (QCA) *General Certificate of Education (GCE) Advanced Subsidiary (AS) and Advanced (A) Level Specifications Subject Criteria for Mathematics* as our *de facto* curriculum. Of course, we acknowledge that these *Subject Criteria for Mathematics* do miss important mathematical topics, notably complex numbers. The European Society for Engineering Education (SEFI) *Working Group on Engineering Mathematics and Education*, Core Curriculum document [12], for example, describes alternative curricula.

Ultimately, in developing our diagnostic tests we identified 103 "core skills". These are grouped into two levels. The top level skills are listed below, together with the codes with which we refer to them.

1. Logic (LOG)
2. Using conventional forms of notation (FOR)
3. Number (NUM)
4. Arithmetic Operations (ART)
5. Algebraic Manipulation (ALG)
6. Estimation (EST)
7. Interpreting Graphs (GRP)
8. Analysis of Diagrams (DIG)
9. Analysis of Word Problems (WRP)
10. Equations (EQN)
11. Functions (FNC)
12. Geometry (GEO)
13. Sequences and Series (SEQ)
14. Logarithms and Exponentials (LGE)
15. Differentiation (DIF)
16. Integration (INT)
17. Vectors (VEC)

These codes are used in the reporting mechanism. We have chosen to subdivide many of these top level skills into subskills. For example, *Equations* (EQN) includes

1. Solve Linear Equations (EQN-LIN)
2. Solve Simultaneous Equations (EQN-SIM)
3. Solve Quadratic Equations (EQN-QUD)
4. Solve Inequations (EQN-INQ)

There are clearly omissions within this list of topics, notably trigonometry and complex numbers. For the purpose of this project we have stopped short of trying to list all topics, or to write questions which assess them. While it is not difficult to list curriculum topics in mathematics, a curriculum is much more than this. In developing an effective diagnostic test we need to understand something of the *style* of questions and the extent to which topics are linked together.

2.1 An example of structuring

Simply listing a topic such as *integration by substitution* gives us little idea of the style of question. Compare the following four versions.

1. Find $\int \sin^3(x)\sin(2x)dx$.
2. Using integration by substitution find $\int \sin^3(x)\sin(2x)dx$.
3. By using the substitution $u = \sin(x)$, or otherwise, find $\int \sin^3(x)\sin(2x)dx$, giving your answer in terms of x.
4. By using the substitution $u = \sin(x)$, or otherwise, show that $\int \sin^3(x)\sin(2x)dx = \frac{2}{5}\sin^5(x) + c$.

On the OCR specimen paper, [13], it is version 3, i.e. giving the explicit substitution but not asking only to verify the answer, which is asked. We believe this is a general trend, with *verification of a given answer* relatively common. At university we would also expect student to be able to recognize which method should be used, and to make choices within the detail of how this method should be employed. Therefore we have a strong preference for questions which instruct the students to *find the answer*. While this may be a change from the conditions under which students are familiar with answering questions, we feel it is justified.

2.2 Proof, reasoning, and problem solving

In addition, the QCA's *Subject Criteria for Mathematics* discuss more general skills such as proving and problem solving, under the heading *Proof*. In particular they claim that "*these requirements should pervade the core content material set out*". We do not discuss the extent to which current UK school examinations do or do not actually implement these specified assessment objectives. However, we do have technical limitations when trying to assess proof with STACK. Therefore, the extent to which we try to assess these specified objectives will be limited, but not completely abandoned. The following example, known as the *Wason selection task*, seeks to test logical reasoning, [14],[15].

> Imagine you have a deck of cards in which every card has one letter on one side and one number on the other. You can see the following four cards
>
> **D** **3** **K** **7**
>
> Turn over the fewest cards to establish the truth of the following statement. "*Every card which has a D on one side has a 7 on the other.*"

2.3 Conditions of the tests

The project sought to develop tests which are suitable to be taken during a single sitting of one hour; with a pencil and paper to hand, i.e. these are not mental tests; with the help of our own formula sheet but without calculators, computer algebra, Wolfram Alpha or similar.

Of course, teachers can use the questions and tests in a variety of ways. Furthermore, if a teacher needs to ensure students take a test under certain conditions, these will need to be invigilated. This said, each question we developed had random versions, formative feedback and full worked solutions reflecting any randomization. Hence, they can be used in a formative or practice setting. If running a diagnostic test these features can be "switched off" by using the appropriate options within STACK.

3 The User Profile

In STACK, each attempt at a question results in three outcomes:

1. Text based feedback;
2. A numerical score/mark;
3. An internal "answer note".

These broadly equate to the formative, summative and evaluative purposes of assessment. Our questions generate a numerical score and full formative feedback. All but minimal feedback is suppressed to enable them to be used in a summative setting. The answer note records the logical outcome regardless of the random version taken. For example, assume we have a question asking a student to find the derivative of an exponential (DIF-

EXP). If a student gets a question correct which we deem needs this skill, they pick up a tag DIF-EXP-TRUE. If a student gets a question incorrect which we deem needs this skill, they pick up a tag DIF-EXP-FALSE.

The reporting software examines these tags to build the profile and match up appropriate follow-up materials. Further work needs to be done to enlarge the range of materials available.

4. Results

The questions were identified by collaborating with colleagues from the University of Birmingham, University of Aston, Loughborough University and the University of Manchester. Existing paper-based tests were used as an initial starting point for many of these questions. For each STACK question we wrote a full worked solution, and the majority of questions were randomly generated from a template. Ultimately the project wrote 87 questions which sought to assess 70 of the 103 sub-skills identified. In addition to this, resources from the mathcentre website were also linked to questions. The questions have been grouped into three categories, depending on the anticipated educational level of students.

A selection of 16 of these questions were taken by 211 first year students in September 2010 at the University of Birmingham. Interestingly, final score correlated most strongly with the *number of questions attempted* indicating that for most students *time taken* for the questions was the most significant factor. However, at least 45 students appeared to have attempted all questions indicating that the test was not unrealistic. This makes analysis of responses to questions later in the test difficult because many students appear not to have attempted them.

Question 3 was taken from [16]

> A university has 6 times as many students as professors. If S represents the number of students and P represents the number of professors, write an equation expressing the relationship between S and P .

When Clement *et al.,* [16], gave the Students-and-Professors Problem to 150 calculus level students, 37% answered incorrectly with 6S=P accounting for two thirds of all errors. For our students the results were better, only 22% of students were incorrect. Interestingly students were *very confident* in their incorrect and correct answers, with 83% of responses indicating a confidence level of more than 90% in the correctness of their answer.

Results from the *Wason selection task*, discussed above, are also interesting. Only 32% of all attempts were correct, i.e. [D,3]. Popular incorrect choices were D only, or [D,7] at 22% each. The only other significant response was [D,3,7] which would confirm the statement but is not minimal. All other combinations amounted to 15% of responses with no individual response attaining more than 5%. With these incorrect answers the full range of confidence was expressed and more than half of responses were more than 75% confident, with 24% of responses being 100% confident. This misplaced confidence warrants further investigation.

Partial fractions, question 7, was a topic which few students could do: of the 211 students there were only 107 *attempts*. 185 students failed to complete this question, only 24 got this correct.

It will be interesting to compare results on this test with marks at the end of the first year to look for any correlations. In future, more detailed analysis of answers could help us to decide which questions are useful in identifying individuals in need of better initial support.

5. Conclusion

Writing a useful diagnostic test for incoming students in the STEM subjects is considerably more involved than identifying a list of curriculum topics and writing question to assess them. This project has succeeded in developing a range of questions which we assembled into quizzes and made available. Following from our use of the diagnostic test we will further evaluate the outcomes, and decide whether the structures put in place for building student profiles is sufficiently valuable to warrant the effort expended in developing a skill classification scheme and tagging all questions individually. In particular we shall evaluate the results of the diagnostic tests in the light of end of year 1 examinations to decide if the results are in any way predictive.

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Dyscalculia in Further and Higher Education

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Abstract

Dyscalculia is one of the newer challenges that face practitioners and researchers, particularly in the post 16 sectors. The focus of this paper is therefore be Further and Higher Education. Dyscalculia is a specific learning difference, which affects the ability to acquire arithmetical skills and an intuitive grasp of numbers. Consideration is given to this and other current definitions, together with a theoretical perspective. The paper also considers the prevalence of dyscalculia, as well as the difficulties dyscalculic students' experience, both in academic life and more generally. The paper highlights DysCalculiUM, a new first-line screening tool for dyscalculia focusing on the understanding of mathematics. The system provides an on-line delivery of the screening tool to identify students at risk with minimal staff input. A Profiler identifies students requiring further investigation. This may take the form of an in-depth interview and referral for further testing. The final section of the paper considers subsequent one-to-one support for students. A case study of a dyscalculic student in Higher Education working with tables of information, percentages and graphs, serves to illustrate some of the ways in which dyscalculic students can be supported on a one-to-one basis.

1. Background

1.1 Towards a Definition

The DSM-IV [1], defines Mathematics Disorder in a person in the following terms: "as measured by a standardised test that is given individually, the person's mathematical ability is substantially less than would be expected from the person's age, intelligence and education. This deficiency materially impedes academic achievement or daily living". One of the key features of this definition is that the mathematical level is significantly lower than expectation. Indeed, Butterworth [2], says: "Most dyscalculic learners will have cognitive and language abilities in the normal range, and may excel in non-mathematical subjects". Another key feature is the impeding of academic achievement and daily living. The National Center for Learning Disabilities [3], says: "Dyscalculia is a term referring to a wide range of life-long learning disabilities involving math… the difficulties vary from person to person and affect people differently in school and throughout life". However, the DSM-IV definition does not expand on what is meant by mathematical ability and this is crucial to our understanding of dyscalculia. The definition is centred on "Mathematics Disorder" and this implies a stable cognitive root, which should not be based on achievement or mastery that is subject to influences of education and environment. Therefore it would seem inappropriate to make assessments through achievement tests, as is often current practice.

The National Numeracy Strategy [4], defines dyscalculia as that which "affects the ability to acquire arithmetical skills. Dyscalculic learners may have difficulty understanding simple number concepts, lack an intuitive grasp of numbers, and have problems learning number facts and procedures. Even if they produce a correct answer or use a correct method, they may do so mechanically and without confidence." The "ability to acquire" is important in that it emphasises acquisition rather than the carrying out of arithmetic procedures. The definition also goes further than the previous one, in that it is more specific about the nature of the mathematical ability: i.e. "difficulty understanding simple number concepts, lack an intuitive grasp of numbers", thus placing understanding at the core of dyscalculia. "A lack of a true comprehension or understanding of maths will be a key characteristic of dyscalculic people" Chinn [5], However, the National Numeracy Strategy [4], continues with the inclusion of "learning number facts and procedures". This would be more indicative of the issues experienced through dyslexia. The dyslexic student is likely to struggle to recall number facts such as times tables and would rely on understanding the mathematics required, rather than well rehearsed procedures learnt by rote or over-learning.

1.2 Prevalence

There is currently no adult data available that gives an estimate of the number identified with dyscalculia. The available data from previous research studies relates to populations of children. Original estimates by Kosc [6] placed this at 6.4% and this is broadly in line with more recent estimates by Butterworth [7], of between 5% and 6%. However, Geary [8], and Desoete [9], both estimate the prevalence of dyscalculia in child populations to be as high as 8%.

1.3 Theoretical Perspectives

Recent advances in neuroscience have greatly increased our understanding of how we conceive number. "An elementary number system is present very early in life in both humans and animals, and constitutes the start-up-tool for the development of symbolic numerical thinking that permeates our western technological societies" [10], Dehaene et al. [11] postulate a triple code theory based on three related neural regions of the brain. The first region is the horizontal segment of the intraperietal sulcus (HIPS) in which numerical quantity is represented and manipulated and which is activated independently of input modality, that is as a digit, word or a collection of items. The second region is the left hemisphere angular gyrus (LAG) in which the verbal processing of numbers takes place and is activated by linguistic rather than quantity processing. It is this area that is responsible for digit naming and learned number facts. The third region is the posterior superior parietal lobule (PSPL) in which visual-spatial processing occurs. The PSPL is activated usually in conjunction with HIPS, during number processing, but is associated with space and time. The PSPL is predominantly within the right hemisphere and the LAG is entirely in the left hemisphere. Thereby, a 3-way system guides and constrains the acquisition of symbolic number skills and, in particular, the HIPS and PSPL are activated during number processing. These are mediated by LAG.

2. A first-line screening tool for dyscalculia

2.1 Development

DysCalculiUM [12], is a first-line screening tool for dyscalculia, developed by Trott and Beacham at Loughborough (due to be published by *Iansyst* in conjunction with *Tribal* in November 2010). The screener focuses on mathematical understanding and has an on-line delivery to identify students at risk of dyscalculia. The tool is designed so as to minimise staff input. A profiler report, given after each student has taken the screening test, identifies those students requiring further investigation (which can be an in-depth interview or referral for further testing). The model for the screener is based on 11 categories, 6 of which are about understanding number and the other 5 relate to common applications. Conceptual understanding of both number and operations relate to activities associated with the HIPS region, while comparisons of number on a visual-spatial plane and with symbolic notation are associated with the PSPL region. Making inferences from given operations requires both manipulation within the HIPS region and visual-spatial processing, thus requiring activations in both HIPS and PSPL. Furthermore, comparisons of number quantity through language would employ the LAG region together with the HIPS region. The model for the DysCalculiUM screening tool is shown in Figure 1.

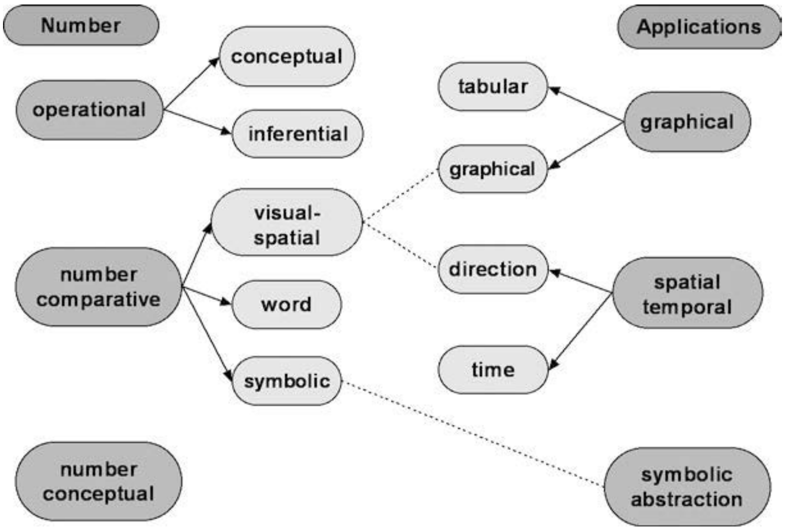

Figure 1: the model for DysCalculiUM, showing the 3 key areas of number, together with their subcategories and the 3 key areas of applications together with their subcategories.

2.2 Trials

The screener has undergone several trials and modifications during its development and the final trials were divided into two parts. In the first, the screener was given to large lecture groups in both Colleges of Further Education and Institutions of Higher Education. The size of each group varied, and the total sample was 504. This provided data for the establishment of "at risk" and "severely at risk" thresholds for the population. The second part involved trials on a one-to-one basis with individuals known to be dyscalculic. This allowed the tool to be verified against a known population. Results from these final trials showed the accuracy of the screener and established reference points for using with future participants. The 8th percentile rank was established as the threshold for "at risk" with the 2nd percentile rank as the threshold for "severely at risk". Of the 51 one-to-one trials with those participants who were known to be dyscalculic, 47 were shown to be "at risk" or "severely at risk" by the DysCalculiUM screening tool. Further information was sought on the four individuals who were not identified by the screener; one assessment report said that the individual was "probably dyscalculic" and the three others were students who were all following science degree courses (biochemistry, physics and computer science). It is likely that these students had developed good coping strategies in order to be able to follow the demands of their courses. These well-developed strategies, that would be frequently repeated during their course, would therefore mediate the score obtained in the screener. This accepted, the screening tool provides a powerful first-line for screening for dyscalculia.

2.3 The Profile

The 11 categories, together with the overall score, are used to create the profile. For each of these, there is an indication of "not at risk", "at risk" and "severely at risk". Guidance is provided on how to interpret the profile and on the subsequent course of action. The procedure is as follows: the learner accesses the DysCalculiUM portal, completes the screener and the results are automatically analysed. The tutor then accesses the portal and reviews the results and profiles, identifying those who are at risk. There can then be a further investigation of the difficulties through an initial tutor-led interview and then by an Educational Psychologist or qualified assessor. A formal identification of dyscalculia can lead to one-to-one support for the learner.

Figure 2 shows an exemplar profile. This profile, for 'Thomas', shows that he is "severely at risk of dyscalculia". This is given, primarily by the overall score indicator, but this is also backed up by the seven categories highlighted as "at risk" or "severely at risk of dyscalculia". The profile further suggested that Thomas had difficulty understanding number concepts and struggled to make numerical comparisons between numbers. It follows that he also had problems with understanding the concept of number operations and in making inferences from them. However, Thomas did not have difficulty with graphical and tabular information, time and spatial directions, which are more visual areas and suggest that Thomas is a visual learner. During the in-depth interview, Thomas revealed that he had

	Severely at risk	At risk	Not at risk
Overall Score:			
1 Number Conceptual			
2 Number Comparative: Word			
3 Number Comparative: Symbol			
4 Number Comparative: Visual Spatial			
5 Graphical			
6 Tabular			
7 Symbolic Abstraction			
8 Spatial Direction			
9 Time			
10 Operational: Conceptual			
11 Operational: Inferential			

Figure 2: DysCalculiUM profile for "Thomas", showing areas of strength, "at risk of dyscalculia" and "severely at risk at dyscalculia"

always had particular difficulties with mathematics, and in school had been placed in the lowest mathematics group. He had achieved outstanding results in many other areas, but had failed to overcome his mathematical difficulties. Consequently, he was very low in confidence and was concerned about the numerical aspects of his course.

3. Dyscalculia: The Social Affects

Although there is still a lack of awareness of dyscalculia as a specific learning difference, number and numerical understanding underpin many of our daily routines, including household budgeting, checking change or telling the time. Thus, dyscalculics face challenges each day. Anxiety, frustration and low self-esteem often result. One dyscalculic student recently said that she always paid with "a purple", meaning a £20 note, thereby ensuring that she had given enough money to cover the cost of her purchases. She could not tender the correct amount nor check her change. She also found it difficult to remember numerous orders when she and her peers went to the cafe. Her embarrassment in this situation led her to stop socialising with her peers and she became increasingly socially isolated. It is therefore important that dyscalculia can be identified effectively so that support can be put in place to enable dyscalculics to reach their full potential and feel more confident in everyday numerical situations. The DysCalculiUM screening tool is a first step in this process.

4. One-to-one Support for Dyscalculia

4.1 Case Study

Following identification, it is important to support the learner in an appropriate way. The following case study will serve to illustrate the ways in which a student with dyscalculia can be supported on a one-to-one basis with the mathematical elements of a course.

'Liam' was a first year student studying transport management and was identified as dyscalculic following initial screening with the DysCalculiUM tool, during his first year at university. Liam had always struggled with understanding basic mathematical concepts and had been placed in the lowest set for mathematics in school. However, he had excelled at other subjects, particularly languages. The initial screening and assessment revealed that he had strengths in several areas, especially verbal reasoning, expressive writing and reading comprehension. At the same time, his dyscalculia resulted in difficulties with conceptual understanding of number and operations as well as the ability to tell the time and understand graphical information. Furthermore, he had particular difficulty with sequencing numbers in the correct order and relating them to a number line and problems with carrying out basic numerical calculations. His work showed that he was unsure of basic operations and frequently used inappropriate strategies.

4.2 Tables of Information

One of the first issues that Liam faced in his course was his difficulty with tables of information. He had to follow a 4 X 4 table showing plane flights to four cities and the number of minutes late arriving. Firstly, he was unsure of the distinction between rows and columns. This was addressed by the use of a picture of a rowing boat that moved in a horizontal direction across the page, while a picture of a Greek architectural column served to indicate the vertical direction implied by the word column. Liam found reference to these images helpful and this also emphasised that he was a visual learner. One of the other issues with the table was that he frequently mixed up the numbers required for any particular calculation (see Table 3a). One example was to calculate the percentage of flights to Brussels that were more than 5 minutes late. In order to identify and focus on the appropriate information, pieces of card were used to cover over the irrelevant information, leaving only the required data visible (see Table 3b). Support sessions then focused on the ideas of fractions and percentages, so that, after some time, he was able to see that 3 out of 12 was a quarter or 25%. Visual representations were also utilised during this process.

Birmingham to	Minutes late (to nearest minute)			
	On time	1 to 5	6 to 10	Over 10
Paris	8	3	1	0
Brussels	6	3	1	2
Munich	4	1	0	0
Dublin	7	1	1	1

Table 3a: showing the frequency of flights from Birmingham to 4 cities, together with the number of minutes late arriving

Birmingham to	Minutes late (to nearest minute)			
			6 to 10	Over 10
Brussels			1	2

Table 3b: with covered cells showing only the frequency of flights from Birmingham to Brussels that were more than 5 minutes late.

4.2 Graphical Representations

Another section of his course required understanding of graphical representations. This was particularly difficult for Liam. The first issue was that he struggled with ordering a sequence of numbers and relating them to a number line. Numbers were written on post-its, one number on each, and Liam was encouraged to put them in order. He also worked on placing them on a number line. When this process was secure, he was able to transfer this to the horizontal axis of a simple linear distance/time graph. Thus, he was able to conclude that time was going forward. However, he was unable to see that the distance was also changing (in this case increasing) simultaneously. The graphs were re-drawn many times without the understanding being present. It was only by chance, that on one occasion he drew the graph with the vertical axis on the right hand side, and he had a "eureka moment". He said: "it's climbing up the wall!" This was a pivotal moment for Liam and, although he often resorted to drawing his graphs "backwards", he was able to move ahead to other aspects of his course with greater confidence. This included correlation and sales forecasting.

Although Liam continued to struggle with the mathematical aspects of his course, he was highly successful in other areas and was able to complete the course.

5. Conclusion

Dyscalculia is one of the newer challenges that face the community of practice. While dyslexia has been the subject of numerous studies and dyslexia support is well established throughout education, dyscalculia is often not recognised or supported. Recent developments in neuroscience have provided an insight into how we conceive and process number; this is invaluable in informing our practice enabling the development of effective screening and support.

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Assessment of Elementary Discrete Mathematics

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Abstract

This paper explores issues in setting objective questions for use in computer-aided assessments (CAA) that form an integral part of a discrete mathematics module taught to our *Foundations of IT* students. We demonstrate how random parameters can be used to make all aspects (scenarios, wording, tables, equations, graphs and diagrams) of question and feedback screens dynamic, so that each will have thousands or millions of realisations. Answer files were analysed to give teaching staff hard data on the facility and discrimination of each question that might consequently be used to inform teaching/assessment strategies. With this aim in mind, we also analysed examination scripts to identify common errors; moreover these can be encoded to improve the CAA by providing responsive feedback that infers from the students' input exactly what error(s) must have occurred. To examine what effect the extensive students' engagement with the CAA has had on their subsequent examination performances, we examined easily-identified indicators for basic skills, and analysed the marks and popularity of whole questions for each of five topics (Number Systems, Sets, Logic, Linear Programming and Graph Theory) for a more synoptic viewpoint on their overall understanding. Results for five years, spanning the introduction of CAA for each topic, show that there has been improvement in some areas, but that students' understanding of Sets is still poor and that Logic has not (yet) improved.

1. Introduction

Discrete mathematics forms an important and significant component of HE curricula in departments of mathematics, computer science and electrical engineering. According to the data tables on UCAS website [1], there was a significant increase in the number of accepted applicants in all three subjects between 2006 and 2009 (Table 1).

	Mathematics	Computer Science	Electrical Engineering
2006	5,412	11,787	4,895
2009	6,916	13,303	5,105

Table 1: The number of accepted applicants in mathematics, computer science and electrical engineering in years 2006 and 2009.

Clearly this represents a considerable 'market' for the development of computer-aided learning (CAL) and computer-aided assessment (CAA) packages, both of which are currently less well developed for discrete mathematics than some other areas such as algebra and calculus. What is more, the majority of students do not take the decision mathematics modules at A level and even then will have had no exposure to some of the topics covered below, especially formal logic. Hence discrete mathematics is generally taught from scratch at university, with few or no prerequisites assumed. This means that discrete mathematics modules will generally have very similar content and level, although they may be taught at different speeds tailored to the cohorts' abilities. Consequently, material designed to assess students with a strong formative element, such as that delivered by the CAA described in this paper, forms a highly-desirable component of any module in discrete mathematics, if only to reinforce the basics of notation and secure fluency in the more mechanistic procedures involved. We therefore recommend that CAA forms part of a blended assessment regime that can effectively span many of the learning outcomes of a module, with written/human-marked components assessing more conceptual or synoptic questions.

We have therefore incorporated CAA within the discrete mathematics syllabus for the target group for this study, namely our *Foundations of IT* students. This 100-strong cohort generally has about 200 UCAS points but has either no mathematics beyond GCSE or rather poor grades at AS or A level. Consequently, no prerequisites are assumed for this second semester module apart from some very basic algebra covered in the preceding semester in their algebra module. The discrete mathematics module comprises Number Systems, Sets, Logic, Linear Programming and Graph Theory, and is assessed by:

- CAA: worth 20% of the module mark, consisting of 5 separate tests each with 5 attempts with the best attempt being counted. This is not invigilated and group work is sanctioned, even encouraged, so that a group of three, say, would have to do the test three times –logged in separately as each group member, and hence get more practice.
- Invigilated unseen written exam: worth 80%, students have 2 hours to answer 4 out of 5 questions, each of equal marks.

The purposes of this paper are:

- to demonstrate that many discrete mathematics topics are, in fact, well suited to objective questioning and hence to CAA,
- to show how effective use may be made of dynamic aspects of questions that incorporate random parameters,
- to see what can be learned from examining the answer files,
- to quantify how effective the embedding of CAA within a module actually is, by examining examination scripts over several years before and after the introduction of CAA for each of the above module topics.

2. CAA Questions

Whilst the principal motivation for the introduction of CAA into a module may be to avoid the tedium involved in human marking, one should seek to capitalise on the full potential of CAA by writing questions that effectively exploit the delivery medium. This process involves the proper formulation of objective questions and associated outcome metadata and is quite unlike the process of writing a problem sheet to be marked by a much more flexible human [2]. For the question setter, this involves a much more clear analysis of the tested and assumed skills needed, and this points towards better assessment overall. However, it is the student that gets the main benefits of repeated practice, each attempt giving full formative feedback. The key to this is to exploit random parameters in all aspects of the questions such as

- Dynamic scenarios. Using scenarios can be effective in 'dressing up' mathematics in a variety of contexts. For example, the linear programming questions pose the same algebraic structure with 'surface effects' arising from scenarios such as optimisation of share returns, zoo visitor hours, hospital treatment regimes … even tobacco addiction rates! The aim is for students to appreciate that they have done similar questions before, by stripping away the context i.e. by thinking at a deeper level of abstraction.
- Dynamic wording. In the translation of English statements into symbolic logic (and vice-versa), all the statements and connectives are randomly selected to form a compound statement (Figure 1).

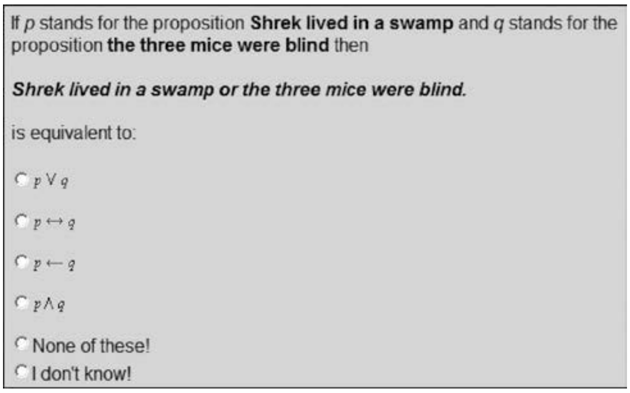

Figure 1: A question on the translation of a simple English statement into symbolic logic, with two propositions and a connective all randomly selected.

- Dynamic tables created on the fly. In the questions on number systems, the size of the table depends on the random numbers chosen, for example how many times successive division yields a non-zero remainder for tables showing Euclid's algorithm, or in logic, the size of the table depends on the number of variables in the proposition (Figure 2).

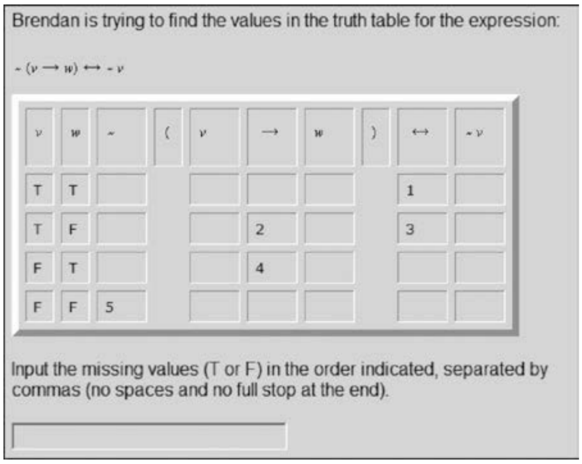

Figure 2: A question on Truth Tables. An external function generates HTML for most of the table, whilst the question coding inputs columns for the actual proposition and the randomly-chosen positions 1-5.

- Dynamic Scalable Vector Graphics (SVG) diagrams, used in the assessment of sets. Randomly chosen numbers appear in Venn diagrams on feedback screens (Figure 3).

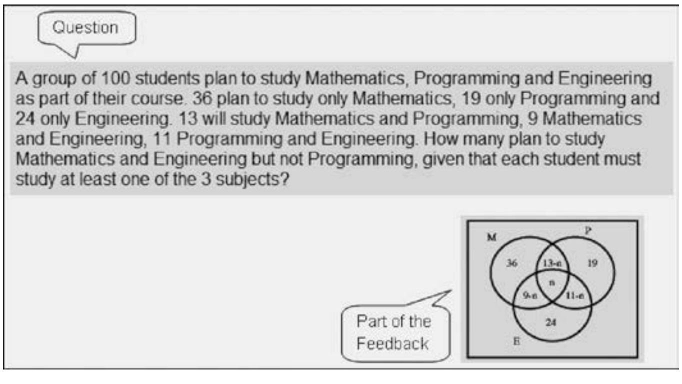

Figure 3: Question text from a Counting Principle question, together with the automatically generated Venn diagram illustrating the distribution of students in the question. An external function generates the SVG code depending on its arguments, for example disjoint sets will be drawn as such.

- Dynamic graphs, present in the linear programming questions. Random parameters are used for objective functions and constraints, giving different graphs each time. The choice of optimum vertex varies and degenerate cases are flagged to the student if they occur.

3. Analysis of answer files

Apart from giving marks to students, the CAA system facilitates easy analysis of answer files. This gives teaching staff a great deal of hard information that would otherwise remain largely anecdotal or based on the experience of the lecturer. Two useful indices are discussed below, with illustrative examples.

3.1 Facility

For the dichotomous marking scheme we generally use, question facility is simply the mean mark. This can be influenced not only by the question content, but also by the question type. Examples of questions students find relatively easy include those which ask them to find a feasible region or translate a simple English statement into symbolic logic (see Figure 1). Both question types here are multi-choice questions, which heavily scaffold the students' thinking by suggesting the form of the answer, encouraging rechecking if they fail to get a displayed answer and rewarding partial knowledge or skills (although 'None of these' can be the correct response). This makes the question much easier which we view positively, especially at the start of a module when one wants to build students' confidence. However, this is not the whole story since difficult multi-choice questions were found in the linear programming assessments. The difficultly here seems to relate to applying sequential tasks correctly, in this case formulating the feasible region and objective function and understanding the roles of their slopes in determining and evaluating the optimal point on the graph. Put simply, there is a lot to do here and a mistake anywhere in the process is likely to be fatal since it may lead to one of the mal-rule-generated distractors or simply to guessing.

Having built confidence (hopefully) one can then invoke more difficult and unscaffolded question types such as True/False/Undecidable (TFU) or Numerical Input (NI) questions where there are no clues and guessing is almost impossible. An example of a TFU question is given in Figure 4, where students have to judge statements on sets algebra. Moreover, they have to get everything right in order to score. Similar comments apply to the question shown in Figure 2.

Let U be the universal set and let A and B be any two distinct non-empty proper subsets of U such that neither is a subset of the other. Moreover $A \cup B \neq U$.

Consider the following table of statements.

If you think the statement is True, type T.
If you think the statement is False, type F.
If you think the statement is Undecidable, type U.

Statement	T, F or U ?
$(A \cap B) \cup \overline{(A \cup B)} = \emptyset$	
$(\bar{A} \cap \bar{B}) \cup (A \cup B) = A$	
$(A \cap B) \cup (A \cap \bar{B}) = A \cup B$	
$\overline{(A \cap B)} \cup \overline{(\bar{A} \cap \emptyset)} = U$	

Remember all inputs must be either T, F or U.
Obviously with this limited number of input choices, a randomly-guessing baboon could do quite well on this question. To gain a mark, you therefore need to get every input correct

Figure 4: A TFU question type on the algebra of sets.

3.2 Discrimination

Discrimination is a statistical measure that compares the students' marks for a question with their overall test marks. An example of a low discrimination question is to input the set $\{x \in N | x<3\}$ explicitly, which almost all got correct since it is mathematically very easy. However, similar questions, but with two inequality signs, cause more problems and only good students can do them, resulting in higher discriminations. Another question which can help to distinguish better students is that shown in Figure 3, where students have to translate the given information into mathematics and recognise what exactly they are asked for. There is a lot of information here; possibly weak students cannot assimilate that amount of information.

4. Analysis of exam scripts

To assess the efficacy or otherwise of the CAA an obvious, but nonetheless useful, comparison to make is that between each CAA test topic and the corresponding marks attained in the exam. However, this masks a number of important issues, for example, not just that students have made a mistake, but what that mistake was and why it was made. For this, some sort of textural analysis of the (large numbers of) exam scripts is needed but this is difficult because of the sheer volume of information. Consequently this paper uses the methodology described in Gill & Greenhow [3, 4] where topic averages and indicators are used. Nevertheless, it is still useful to look at specific common errors.

4.1. Errors

Very often it is difficult or even sometimes impossible to understand students' thinking behind given answers but, for example, some common errors on exam scripts for questions on sets can be categorised as:

- conceptual:
  - confusing union, intersection and difference,
  - thinking that if two non-empty sets don't have any common elements then the difference between them is an empty set.
- notational:
  - not using brackets or using round brackets when listing set's elements,
  - using wrong notation for an empty set e.g. {ø},
  - using symbols $\cup$, $\cap$ and \ inconsistently.
- terminological:
  - even numbers (not including negative numbers and 0),
  - prime numbers (not including 2; including 1 and negative numbers),
  - perfect squares (not including 0 and 1; including negative numbers).

What is good about such clearly-defined errors is that we can program them into the questions; we can also include them in the teaching, placing more emphasis on issues that are confusing students. This has the potential to increase the quality of teaching in the long run, especially if errors can be codified and shared between teaching staff. What might be needed is a commonly-accepted 'taxonomy of errors', perhaps along the lines of Dawkins [5] or Schechter [6], but this seems to be missing.

4.2 Indicators

Exam scripts have also been analysed using four (easily-identifiable) indicators that were selected by the authors because they are necessary, but not sufficient, for deeper student learning. The purpose of this was to find out if the CAA helps students to improve, see Figure 5. During academic years finishing in 2006, 07 and 08 students didn't have access to CAA and were taught by another staff member, Greenhow taking over in 2008/09 and using CAA (but with no other significant changes to syllabus, exam standard, time allocated to each topic or teaching style). There are two gaps in Figure 5, as it was difficult to identify two of the indicators on the exam in May 2007.

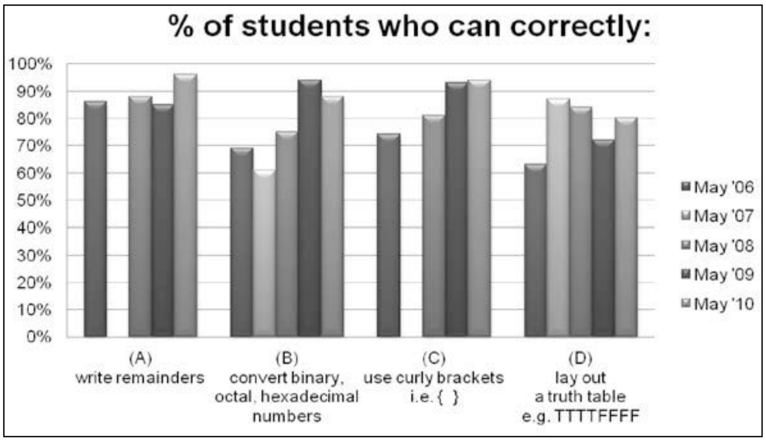

Figure 5: A year-by-year comparison of students' ability to: (A) write remainders; (B) convert binary, octal and hexadecimal numbers; (C) use curly brackets in sets; (D) correctly lay out a truth table.

Overall, these indicators suggest a positive message about CAA where indicators (B) and (C) show significant improvement when CAA was used, and (A) shows that they have improved in the last year.

4.3 Topic comparison of the exam results

Of course, indicators can show only that students have learned one skill, e.g. a simple syntax or notation, but this does not guarantee that they will be able to answer questions that require more synthesis. To examine this we look at the marks awarded for each question/sub-question on a topic-by-topic basis (see Figure 6).

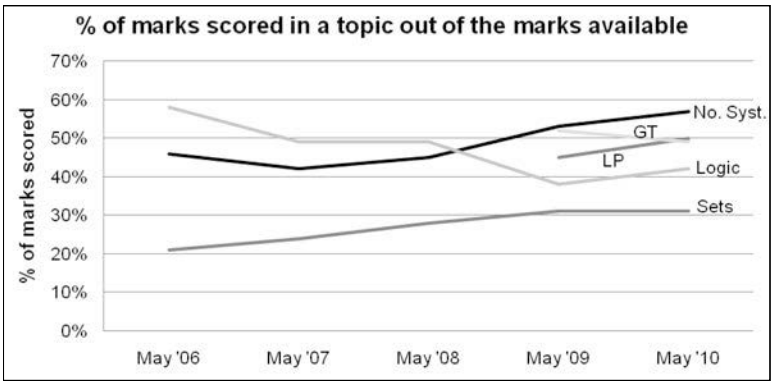

Figure 6: A year-by-year comparison of the % of marks scored in a topic out of the marks available.

<table>
<tr><th></th><th></th><th>Number Systems</th><th>Sets</th><th>Logic</th><th>Linear Programming</th><th>Graph Theory</th></tr>
<tr><td rowspan="3">2008/09</td><td>Exam Results</td><td>76%</td><td>38%</td><td>59%</td><td>81%</td><td>82%</td></tr>
<tr><td>Popularity</td><td>64%</td><td>62%</td><td>49%</td><td>39%</td><td>43%</td></tr>
<tr><td>No. of times q's were run in CAA</td><td>1252</td><td colspan="2">930</td><td>730</td><td>968</td></tr>
<tr><td rowspan="3">2009/10</td><td>Exam Results</td><td>72%</td><td>33%</td><td>60%</td><td>71%</td><td>80%</td></tr>
<tr><td>Popularity</td><td>75%</td><td>65%</td><td>64%</td><td>25%</td><td>47%</td></tr>
<tr><td>No. of times q's were run in CAA</td><td>1743</td><td>1407</td><td>1562</td><td>1166</td><td>1358</td></tr>
</table>

Table 2: For each topic, this table shows the popularity of corresponding exam question and the number of attempted CAA questions in that topic. Topics are presented in the order in which they were taught.

Not much can be concluded about Linear Programming (LP) and Graph Theory (GT), as they have been just introduced and so there is no previous results to compare with. However, results for the other three topics can be compared. Sets and Number Systems (No.Syst.) show an increase in average mark. Sets marks are still bad, but at least are going in the right direction. Logic is different: CAA does not appear of benefit, or is even making things worse, despite the fact that Table 2 shows students' engagement with the logic questions was comparable with that of other topics. It is problematic to explain this result and we await data from future years.

Table 2 clearly shows there has been rise in the engagement with CAA, but that Sets is still a problem. Of equal interest is the rank ordering of the exam results and the popularity of these questions. It seems to be related to when in a module you teach things. If students stay to the end of the course they can do Linear Programming and Graph Theory questions well, but they are not popular since only those students can even attempt them. Students that do not attend to the end are then forced to at least attempt questions on earlier topics, and often do them badly (as for Sets).

5. Conclusions

This paper provides evidence that setting objective questions for topics in discrete mathematics is feasible and that CAA can exploit the possibilities offered by including random parameters within questions scripts. Analysis of the answer files shows that there is a clear engagement with CAA and our perception is that it is popular with students. However, this may be more to do with students seeking to maximise their marks, rather than their understanding of each topic area. We therefore examined exam scripts, partly to identify what errors were being made, but also to measure the efficacy of learning via CAA when embedded in an otherwise traditional module. Here the situation is far from clear, with CAA showing benefits for two out of 4 indicators and in 2 out of 3 topics, but making things worse for Logic. We do not understand these, probably quite subtle, effects and further data will be needed, possibly with control groups in other HEIs. Nevertheless, what is clear is that lecturers can gather hard information from answer files on errors and embed them in teaching and assessment. Another clear message is that only a self-selected group of the better students bother with topics taught later in the module, but that they do well on those topics.

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