Problematic concepts in 1st year Mathematics

Assessment for Learning: Resources for First Year Undergraduate Mathematics Modules
Caitríona Ní Shé

Funded by:
Mathematics Problem
- transition from rote learning to independent thinking
- students grapple with 1st year undergraduate mathematics

National Forum for the Enhancement of Teaching and Learning
- shape the delivery of an outstanding teaching and learning experience at third level in Ireland
  - Building digital capacity
  - Teaching for Transitions
Assessment for learning

- Improve the teaching and learning experience of first year undergraduate mathematics modules
  - identify mathematical topics and concepts that are problematic
  - develop online activities and tasks to promote understanding of these concepts
  - and evaluate the effectiveness of these resources
  - provide the resources to all Higher Education Institutes (HEI’s) in Ireland
Identifying topics and concepts that First Year students struggle with in Mathematics

Caitríona Ní Shé
Outline of presentation

- Survey & Outcome
- Problem Topics & Concepts
  - Likert item analysis
  - Open ended question coding
- Resources
  - What and why used
- Next Steps
Surveys: Spring 2015 – end of 1st year

Identify problematics concepts AND useful resources
- Questionnaire developed by team

Student Survey (460)

<table>
<thead>
<tr>
<th>Institution</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIT</td>
<td>48</td>
</tr>
<tr>
<td>DCU</td>
<td>204</td>
</tr>
<tr>
<td>DKIT</td>
<td>101</td>
</tr>
<tr>
<td>MU</td>
<td>107</td>
</tr>
</tbody>
</table>

Lecturer Survey (33)

<table>
<thead>
<tr>
<th>Institution</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>IT</td>
<td>16</td>
</tr>
<tr>
<td>UNIVERSITY</td>
<td>17</td>
</tr>
</tbody>
</table>
23 different mathematical question types
  ◦ Understand and Do

14) Finding limits of functions using rules of limits

<table>
<thead>
<tr>
<th></th>
<th>Strongly Agree</th>
<th>Agree</th>
<th>Neutral</th>
<th>Disagree</th>
<th>Strongly Disagree</th>
</tr>
</thead>
<tbody>
<tr>
<td>I understand the ideas in questions like this.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I am able to do questions like this.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example of Question

14. Finding limits of functions using rules of limits

Example: Use the rules of limits to evaluate

$$\lim_{x \to 2} \frac{3x^2 - 4x + 9}{x + 5}.$$
Open ended Questions

- Student survey had 7 questions
  - Topics most difficult and most easy
  - Resources they used and why
  - Gaps and how they should be filled

- Lecturer survey had 7 questions
  - Concepts, procedures and tasks difficult
  - Resources they recommend and why
  - Gaps and how they should be filled
Surveys: What did we find out?

Concepts and Topics
- Prior mathematics level as a predictor
- Student and Lecturers differing views
- Themes emerging

Resources
- Online videos and websites etc
- Prescribed books & Handouts
- Student reliance on examples and solutions
# Student Background Data

<table>
<thead>
<tr>
<th>Category</th>
<th>Gender</th>
<th>Mature</th>
<th>Leaving Cert Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Males</td>
<td>293 (65%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Females</td>
<td>160 (35%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non Mature</td>
<td></td>
<td>368 (84%)</td>
<td></td>
</tr>
<tr>
<td>Mature</td>
<td></td>
<td>69 (16%)</td>
<td></td>
</tr>
<tr>
<td>Higher</td>
<td></td>
<td></td>
<td>282 (64%)</td>
</tr>
<tr>
<td>Ordinary</td>
<td></td>
<td></td>
<td>142 (32%)</td>
</tr>
<tr>
<td>Foundation</td>
<td></td>
<td></td>
<td>5 (1%)</td>
</tr>
<tr>
<td>Did not take</td>
<td></td>
<td></td>
<td>15 (3%)</td>
</tr>
<tr>
<td>Total Responses</td>
<td>453</td>
<td>437</td>
<td>444</td>
</tr>
</tbody>
</table>
Leaving Certificate Mathematics Level

Mathematics Level per institute

<table>
<thead>
<tr>
<th>Institute</th>
<th>Higher</th>
<th>Ordinary</th>
<th>Foundation</th>
<th>Did Not Take</th>
<th>No Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DCU</td>
<td>150</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DKIT</td>
<td></td>
<td></td>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MU</td>
<td>100</td>
<td></td>
<td>50</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Number of Students
Problem Topics and Concepts

Students & Lecturers
Students have few problems!
Students have few problems!

Almost 20% disagreement

Q13: Finding limits of functions using graphs
Q14: Finding limits of functions using rules of limits
Q21: Finding stationary points
Q22: Optimisation (max/min) problems
Q23: Graph sketching using derivatives
No questions with 20% disagreement
Approximately 20% disagreement
Q4: Powers – using laws of indices
Q5: Logs – using the laws of logarithms to simplify expressions
Q6: Using the connections between logs and exponents
Q13: Finding limits of functions using graphs
Q14: Finding limits of functions using rules of limits
Q16: Deciding whether a function is continuous or not
Q21: Finding stationary points
Q22: Optimisation (max/min) problem
Q23: Graph sketching using derivatives
Higher Level Versus Ordinary Level

- Chi-Squared test to compare Higher and Ordinary level responses to the Likert items
- Reduced to 3 point scale agreement, neutral, disagreement

\[ p = 0.004 \]

- There is a dependency between Leaving Certificate Mathematics Level taken by student and their rating of their ability to both Understand and Do the 23 questions types in the Likert items
What does Likert survey tell us?

- Overall students report very few problems

- Students who had taken Ordinary Level mathematics perceive they have more difficulties than those who took Higher Level

- Problem areas are logs, exponents, limits of functions, graphing and advanced 1st year calculus
Open ended questions analysis

- Imported to Nvivo
- Students
  - Topics most difficult
  - and topics most easy
- Coded to select most frequent
## Coding in Nvivo – difficult topics

### No. of responses containing these codes (414)

<table>
<thead>
<tr>
<th>Topic</th>
<th>No. of Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebra and Linear</td>
<td>17</td>
</tr>
<tr>
<td>Algebra</td>
<td>32</td>
</tr>
<tr>
<td>Logs (without diff or integration)</td>
<td>61</td>
</tr>
<tr>
<td>Trigonometry</td>
<td>32</td>
</tr>
<tr>
<td>Optimisation AND limits</td>
<td>4</td>
</tr>
<tr>
<td>Optimisation</td>
<td>31</td>
</tr>
<tr>
<td>Graphing Functions</td>
<td>38</td>
</tr>
<tr>
<td>Functions (func*)</td>
<td>47</td>
</tr>
<tr>
<td>Limits (lim*)</td>
<td>38</td>
</tr>
<tr>
<td>Applications</td>
<td>6</td>
</tr>
<tr>
<td>Integration and/or Differentiation of Trig functions</td>
<td>13</td>
</tr>
<tr>
<td>Calculus (without differentiation and integration)</td>
<td>12</td>
</tr>
<tr>
<td>Logarithmic differentiation</td>
<td>11</td>
</tr>
<tr>
<td>Integration AND Differentiation</td>
<td>46</td>
</tr>
<tr>
<td>Differentiation</td>
<td>89</td>
</tr>
<tr>
<td>Integration AND ideas AND methods OR method</td>
<td>9</td>
</tr>
<tr>
<td>Integration AND parts</td>
<td>27</td>
</tr>
<tr>
<td>Integration AND parts AND ideas</td>
<td>2</td>
</tr>
<tr>
<td>Integration AND methods OR method</td>
<td>34</td>
</tr>
<tr>
<td>Integration AND ideas</td>
<td>22</td>
</tr>
<tr>
<td>Integration</td>
<td>159</td>
</tr>
</tbody>
</table>
Higher versus Ordinary

% of student who responded and mentioned these topics
(414, 251 HL, 134 OL)
Open Ended: Easy Topics

% of student with mathematics level who responded and mentioned these topics (407)

<table>
<thead>
<tr>
<th>Topics mentioned most frequently by students</th>
<th>Higher level completed</th>
<th>Ordinary Level completed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Differentiation</td>
<td>40.00</td>
<td>25.00</td>
</tr>
<tr>
<td>Integration</td>
<td>15.00</td>
<td>5.00</td>
</tr>
<tr>
<td>Logs</td>
<td>10.00</td>
<td>5.00</td>
</tr>
<tr>
<td>Limits</td>
<td>5.00</td>
<td>2.00</td>
</tr>
<tr>
<td>Graphs</td>
<td>10.00</td>
<td>5.00</td>
</tr>
<tr>
<td>Functions</td>
<td>10.00</td>
<td>5.00</td>
</tr>
</tbody>
</table>
A number of themes
- Prior mathematical background
- Method selection
- Applications
- Visible
Topics cause difficulty and why

“Optimization and graphing functions. It was one of the topics that was least covered in secondary school, therefore coming into university, studying maths at a higher level was a bit difficult to grasp. The formulas and when to use the right ones was the difficulty”
Student Comment: Leaving Cert

- Topics most easy and why

“most easy were topics I covered in Leaving cert like: linear equations, graphs, algebra functions, quadratic equations”
Student Comment: Applications

- Topics cause difficulty and why

“When we done concave up and concave down graphs, it was very hard to apply yourself to a concept which seemed so useless, it didn't seem to have any practical use so it was hard to apply yourself to it and the Planes section from linear algebra”
Student Comment: Visible

- Topics most easy and why

Interpreting graphs and base functions because the date is visible and relatable to application and its easier to understand as a result
Lecturer viewpoint

Concepts most difficult

- Basic Algebra: 19
- Functions and Graphing and Limits: 15
- Calculus: 13
- Geometry: 2
- Linear Algebra: 8
- Probability and Statistics: 5
- Trigonometry: 4
Lecturer versus student recoding

Concepts most difficult

%students responses

Basic Algebra | 19
Basic Arithmetic | 9
Calculus | 15
Functions and... | 13
Geometry | 8
Linear Algebra | 5
Probability and... | 4
Trigonometry |
Lecturer Further Breakdown

Lecturers Survey: 13 specifically reference logs, indices or exponents.

- 42% (13/32) of lecturers indicate that their students experienced problems with these

- Only 18% (74/414) of students mentioned these
Lecturer Comments: Logs

“Logarithms – general understanding and how to use rules to solve equations; application to experimental laws”

“In the case of Logs, once the student understood the concept of adding the indices of was the same as multiplication of two or more numbers, their engagement with the topic increased”
“I found **logs** and limits the most difficult as the ideas are hard to understand. Also some integration towards the end of the year was challenging as the methods were hard to start off”

“mostly ideas of topic confuse me most. Most difficult was algebraic manipulation of rules of **indices**. Without knowing these it was hard to do a lot of the maths”
Students Found Useful

Number of students

- Books
- Handouts
- Videos
- Websites
- Other
Students Found Useful

- Recommended texts (DCU and MU)
- Lecture, class or tutorial handouts
  “the notes from the support centre were excellent, they summarised the course in an approachable but concise manner, making it easier for looking over just before the exams “
- Videos and Websites
  - Khan Academy, Wolfram Alpha, YouTube, VLE
    “Khan academy is fantastic. The guy who does the video works everything out in simple steps and explains everything as he goes along “
Lecturer Found Useful

Number of Lecturers

- Handouts: 11
- Websites: 15
- Books: 8
- Maths Support: 6
Lecturer Found Useful

- Mathstutor, mathcentre, khan academy, geogebra, VLE

“For weak students the Kahn academy has been good. – We use MathXL as an online assessment tool for homework, providing instant feedback and this has been somewhat useful”

- Handouts
  “Handouts in class seem to help hold attention”
Advise on Resources: Students

- 252 responses

![Bar chart showing the number of students for different resources.

- Print Based: 190
- Videos: 160
- Websites: 220
- Other: 70

Legend:
- Print Based
- Videos
- Websites
- Other

Number of students
Advise on Resources: Students themes

- Solutions, steps and examples
  - video tutorials of different styles of questions
  - Videos of someone going through past exams
  - Examples of different questions to be given on line with step by step solutions and methods that people of all levels could relate to.

- Printed steps of class procedures
Conclusion

Students & Lecturers
Next Steps: Topics Identified

Basic Algebra
- Transposition of formulae
- Logs & Exponentials
- Functions
- Limits
- Logs
Currently developing resources:
- An inventory of useful existing resources;
- Moodle Courses and assessments using Khan Academy materials;
- Online ‘lessons’ and quizzes designed in Moodle;
- Student screencast continuous assessment projects;
- Audience Response system
- A suite of interactive tasks.
Next Steps: Evaluation

- Autumn and Spring 2015/2016 Academic year
  - Survey
  - Focus group interviews
  - Task based interviews
Other relevant projects

- UCD – mathematical difficulties as experienced by students in a Mathematics Support Centre

- UCC and CIT – Transitioning to e-assessment in Mathematics education
The End

Thank you