
Investigating students' difficulties with differential equations in physics.
Thanks for your time.
Questions?


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Project overview

Mathematics

- Classification
- Solution techniques: exact
- Theoretical aspects

Physics

- Solution techniques: Approximations
- Interpretations
- Applications
- Modelling

Key Research Question

What is the precise nature of the difficulties encountered by physics students in using differential equations, and how might these be addressed.

Aims of the project

Identify and understand the difficulties

Develop an intervention

Theoretical perspective

APOS Theory

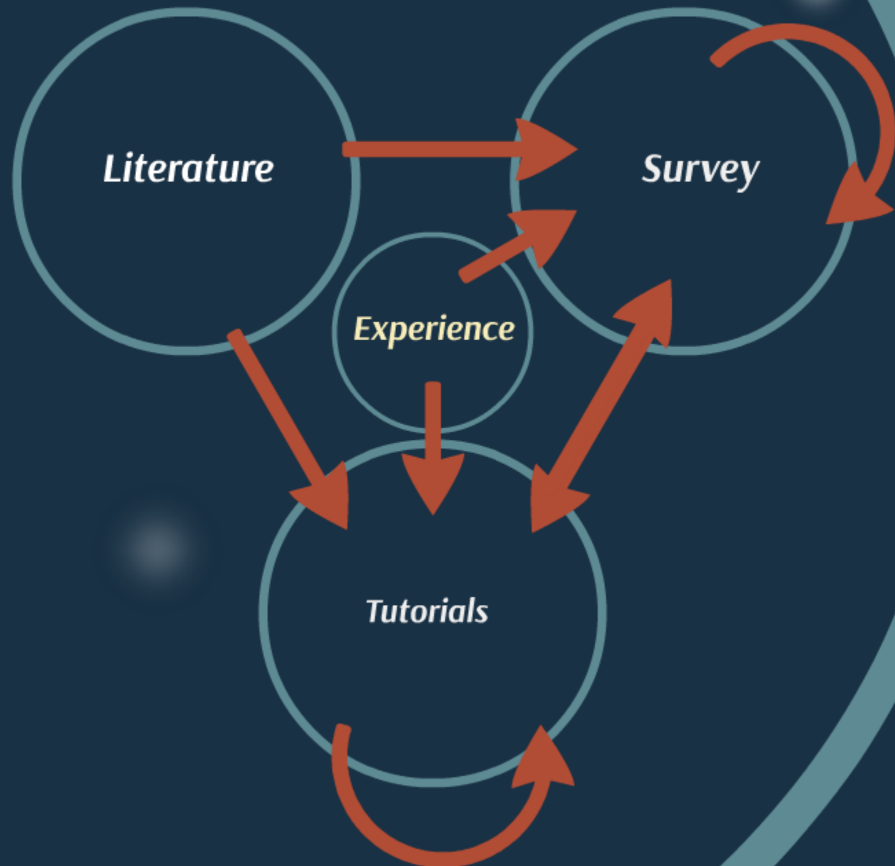
Action
Process
Object
Schema

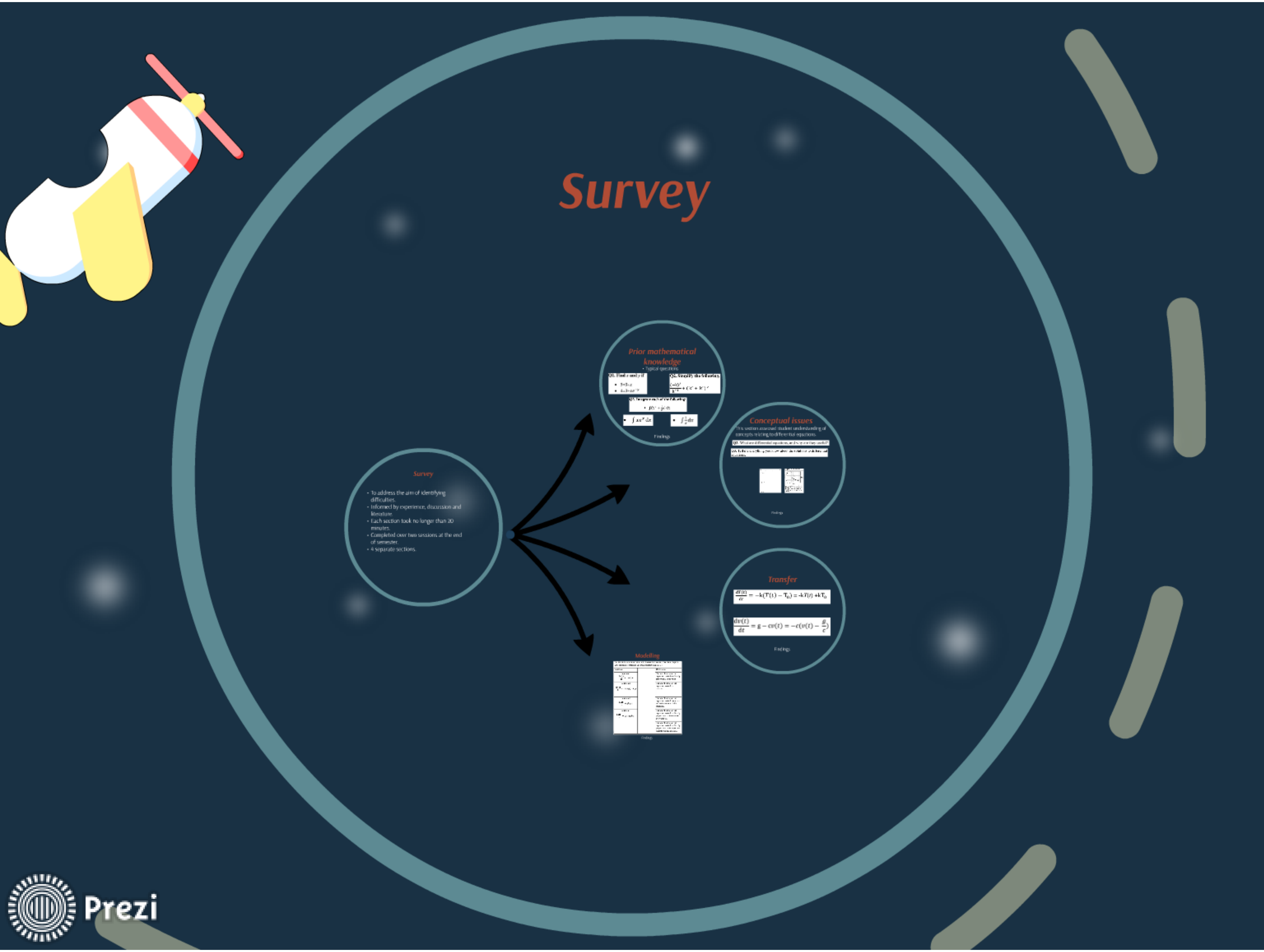
An attempt to extend Piaget's work on reflective abstraction to collegiate mathematics learning.

Methodology

Aims of project

- Identify and understand the difficulties
- Develop an intervention




Survey
Survey
To address the aim of identifying difficulties.
Informed by experience, discussion and literature.
Each section took no longer than 20 minutes.
Completed over two sessions at the end of semester.
4 separate sections.
Prior mathematical knowledge
Typical questions
Findings
Conceptual issues
This section assessed student understanding of concepts relating to differential equations.
Findings
Transfer
$\frac{dT(t)}{dt} = -k(T(t) - T_0) = -kT(t) + kT_0$
$\frac{dv(t)}{dt} = g - cv(t) = -c(v(t) - \frac{g}{c})$
Findings
Modelling
Findings
Prezi

Survey

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Prior mathematical knowledge

- Typical questions

Q1. Find *x* and *y* if

- $5=3+x$
- $4=3+xe^{-y}$

Q2. Simplify the following.

$$\frac{(-k)^2}{k^{-1}} + (k^2 + k^1)^2$$

Q3. Integrate each of the following;

- $\int (x^2 + \frac{1}{x^3})\,dx$
- $\int xe^x\,dx$
- $\int \frac{1}{x}\,dx$

Findings

This sec
concep

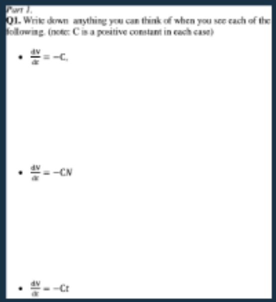
Q2. What

Conceptual issues

This section assessed student understanding of concepts relating to differential equations.

Q2. What are differential equations, and why are they useful?

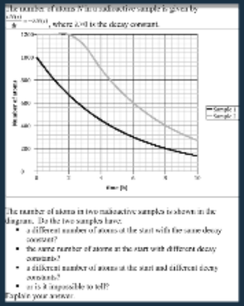
Q3. Tell me everything you know about the solutions to differential equations.



Part 1.

Q1. Write down anything you can think of when you see each of the following. (note: C is a positive constant in each case)

- $\frac{dN}{dt} = -C,$
- $\frac{dN}{dt} = -CN$
- $\frac{dN}{dt} = -Ct$



The number of atoms in two radioactive samples is shown in the diagram. Do the two samples have:

- a different number of atoms at the start with the same decay constant?
- the same number of atoms at the start with different decay constants?
- a different number of atoms at the start and different decay constants?
- or is it impossible to tell?

Explain your answer.

Findings

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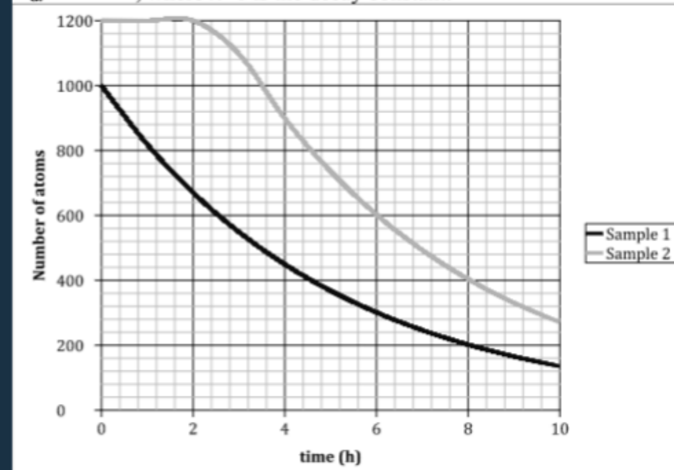
The number of atoms N in a radioactive sample is given by $\frac{dN(t)}{dt} = -\lambda N(t)$, where $\lambda > 0$ is the decay constant.



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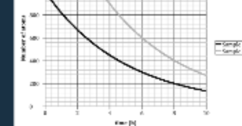

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Explain your answer.

Findings

Transfer

$$\frac{\mathrm{d}T(t)}{\mathrm{d}t} = -\mathrm{k}(T(t) - \mathrm{T_0}) = \text{-}\mathrm{k}T(t) + \mathrm{kT_0}$$

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} = \mathrm{g} - \mathrm{c}v(t) = -c(v(t) - \frac{g}{c})$$

Findings

Prezi

Modelling

Q1. Match the equations below with the text that best describes them. Explain your reasoning. (note: k is a positive constant in each case)

Equation		Description
Equation 1 $$\frac{dA(t)}{dt} = -kA(t)$$		The rate of change of the dependent variable is directly proportional to its value.
Equation 2 $$\frac{dB(t)}{dt} = -k(B(t) - B_0)$$		The rate of change of the dependent variable is constant.
Equation 3 $$\frac{dC(t)}{dt} = k^2 + 4$$		The rate of change of the dependent variable increases as the dependent variable increases.
Equation 4 $$\frac{dE(t)}{dt} = 14 + kE^3(t)$$		The rate of change of the dependent variable is directly proportional to the square of two variables.
		The rate of change of the dependent variable is directly proportional to the dependent variable minus a constant.

Tutorials

modelling and understanding

Modelling with first order ODEs
-Experiments
-build from scratch

Meaning of a devative tutorial an example of a tutorial designed to improve conceptual understanding.

Prezi

modelling and understanding

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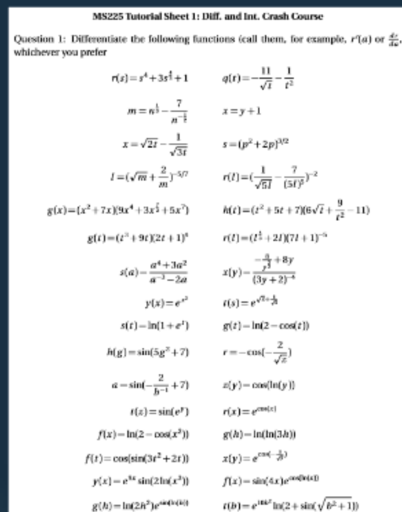
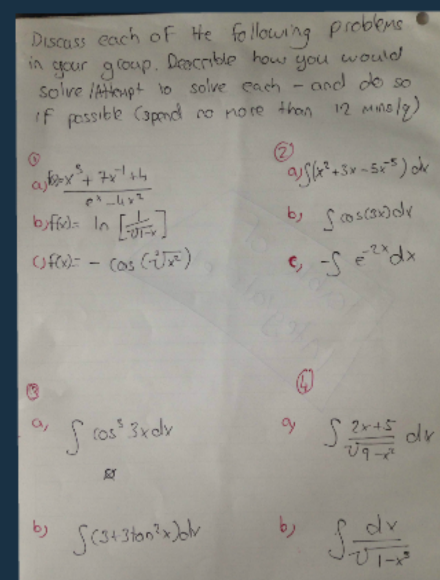


MS225 Tutorial Sheet 1: Diff. and Int. Crash Course

Question 1: Differentiate the following functions (call them, for example, $r'(a)$ or $\frac{dr}{da}$, whichever you prefer

$r(s)=s^4+3s^{\frac{4}{3}}+1$	$q(t)=-\frac{11}{\sqrt{t}}-\frac{1}{t^2}$
$m=n^{\frac{1}{3}}-\frac{7}{n^{-\frac{1}{2}}}$	$x=y+1$
$x=\sqrt{2t}-\frac{1}{\sqrt{3t}}$	$s=(p^2+2p)^{3/2}$
$l=(\sqrt{m}+\frac{2}{m})^{-5/7}$	$r(l)=(\frac{1}{\sqrt{5l}}-\frac{7}{(5l)^3})^{-2}$
$g(x)=(x^2+7x)(9x^4+3x^{\frac{2}{3}}+5x^7)$	$h(t)=(t^2+5t+7)(6\sqrt{t}+\frac{9}{t^2}-11)$
$g(t)=(t^3+9t)(2t+1)^8$	$r(l)=(l^{\frac{1}{2}}+2l)(7l+1)^{-5}$
$s(a)=\frac{a^4+3a^2}{a^{-3}-2a}$	$x(y)=\frac{-\frac{6}{y^3}+8y}{(3y+2)^{-4}}$
$y(x)=e^{x^2}$	$t(s)=e^{\sqrt{s}+\frac{1}{s^3}}$
$s(t)=\ln(1+e^t)$	$g(t)=\ln(2-\cos(t))$
$h(g)=\sin(5g^7+7)$	$r=-\cos(-\frac{2}{\sqrt{z}})$
$a=\sin(-\frac{2}{b^{-1}}+7)$	$z(y)=\cos(\ln(y))$
$t(z)=\sin(e^y)$	$r(x)=e^{\cos(x)}$
$f(x)=\ln(2-\cos(x^2))$	$g(h)=\ln(\ln(3h))$
$f(t)=\cos(\sin(3t^2+2t))$	$x(y)=e^{\cos(-\frac{1}{y^3})}$
$y(x)=e^{5x}\sin(2\ln(x^2))$	$f(x)=\sin(4x)e^{\cos(\ln(x))}$
$g(h)=\ln(2h^2)e^{\sin(\ln(h))}$	$t(b)=e^{10b^2}\ln(2+\sin(\sqrt{b^2+1}))$



Discuss each of the following problems in your group. Describe how you would solve/Attempt to solve each - and do so if possible (spend no more than 12 mins/q)

① a) $f(x)=\frac{x^5+7x^{-1}+4}{e^x-4x^2}$

b) $f(x)=\ln\left[\frac{1}{\sqrt{1-x}}\right]$

c) $f(x)=-\cos(\sqrt[3]{x^2})$

② a) $\int(x^2+3x-5x^{-5})\,dx$

b) $\int\cos(3x)\,dx$

c) $-\int e^{-2x}\,dx$

③ a) $\int\cos^5 3x\,dx$

b) $\int(3+3\tan^2 x)\,dx$

④ a) $\int\frac{2x+5}{\sqrt{9-x^2}}\,dx$

b) $\int\frac{dx}{\sqrt{1-x^3}}$

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$g(t)=(t^3+9t)(2t+1)^4$	$r(l)=(l^{\frac{4}{3}}+2l)(7l+1)^{-5}$
$s(a)=\frac{a^4+3a^2}{a^{-3}-2a}$	$x(y)=\frac{-\frac{8}{y^{\frac{3}{2}}}+8y}{(3y+2)^{-4}}$
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①

a) $f(x) = \dfrac{x^3 + 7x^{-1} + 4}{e^x - 4x^2}$

b) $f(x) = \ln\left[\dfrac{1}{\sqrt{1-x}}\right]$

c) $f(x) = -\cos\left(\sqrt[3]{x^2}\right)$

②

a) $\int (x^2 + 3x - 5x^{-5})\, dx$

b) $\int \cos(3x)\, dx$

c) $-\int e^{-2x}\, dx$

③

a) $\int \cos^3 3x\, dx$

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④

a) $\int \dfrac{2x+5}{\sqrt{9-x^2}}\, dx$

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