Reflections on a Highly Successful Foundation/Access Mathematics Course

September 1, 2014

Outline

- The NUI, Galway "Access" programmes
- Teaching Mature Students
- Ideas and Suggestions

The Access Foundation Courses for Mature Students

To quote from the access office website:

"The Access Course is a 1 year part-time evening course that aims to provide mature students with the opportunity to prepare, personally and academically, for an undergraduate course of full-time study of at least three years duration at NUI, Galway. The course is designed to meet the learning needs of the adult student; to provide individual attention and assistance where appropriate; and to provide financial support, contingent upon funding. The Access course is designed specifically for people who may not have the conventional educational requirements and/or who come from socio-economic backgrounds that are under represented at third level. The aims of the course is to enable students to acquire the skills, knowledge and confidence to compete on an equal footing with those students who enter NUI, Galway through conventional entry channels."

History

- In the 1990's there was an increasing number of applications from mature students who wished to enter degree programmes in Science and Engineering.
- Very few (one or two) would be accepted–usually nothing was known about these applicants.
- The government was putting pressure on the university to accept more mature students
- Ther deans of science and engineering devised the course as a way of assessing potential mature students and equipping them with necessary skills.

- Students who pass ($\geq 40\%$) both the science and maths exam are guaranteed entry to general science, and may apply for the denominated science degrees.
- Students who pass the science exam and get $\geq 60\%$ in the maths exam may apply for an engineering degree, and usually they are admitted.
- Students who pass ($\geq 40\%$) both the science and maths exam are also guaranteed entry into various science and engineering degrees in a “polytechnic type" institution in Galway. Few do this however.

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- that's a lot of marking!!

The Final Exam

- Three Hours
- Seven Questions, do any five. (This varies slightly from year to year).

Question One

1. Simplify:

$$(x - 2y - 3z)(x + 2y + 3z)$$

2. 1. Show that 4 is a root of $w^3 - 9w^2 + 6w + 56 = 0$.
   2. Hence, or otherwise, solve the equation $w^3 - 9w^2 + 6w + 56 = 0$.

3. Make r the subject of: $l = \sqrt{\frac{\pi t r^3}{4}}$

4. Decompose the following expressions into partial fractions:

$$\text{(i)}\ \frac{11x - 21}{x^2 - 4x + 3} \qquad \text{(ii)}\ \frac{4x^2 - 21x + 21}{(x - 11)(x^2 + x + 5)}$$

Question Two

1. Let

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 2 & -1 & 4 \\ 5 & 1 & 1 \end{pmatrix}$$

    1. Find A^*, the adjoint of A;
    2. find $|A|$, the determinant of A;
    3. hence find A^{-1}, the inverse of A.

2. Solve the system of equations:

$$\begin{aligned} x + \quad\quad z &= 4 \\ 2x - y + 4z &= 8 \\ 5x + y + z &= 18 \end{aligned}$$

Question Three

1.
   1. Simplify: $(x \sin\theta + y\cos\theta)^2 + (x\cos\theta - y\sin\theta)^2$
   2. State the amplitude and the period of the trigonometric function
   $$2\cos\left(3\theta + \frac{\pi}{4}\right)$$
   and hence *sketch* its graph over the interval $[0, 2\pi]$.
   3. Maria stands at a point, p, on a straight line between two hills. The angle of elevation to the top of the higher hill is $30°$, and the angle of elevation to the top of the lower hill is $20°$. She climbs to the top of the higher hill, and finds that the angle of depression to the top of the lower hill is $15°$. If the horizontal distance from the point p to the top of the higher hill is 1000 metres, what is the horizontal distance between the tops of the two hills?

Question Four

1. Find both roots of $z^2 - 2z + 2 = 0$, and illustrate them on an Argand diagram.
2. Let $z_1 = 3 - 2i$, $z_2 = 2 + 2i$ and $z_3 = -1 + 3i$. Express $\frac{z_1 z_2}{z_3}$ in *both* Cartesian and polar coordinates, (i.e. in the forms $a + bi$ and $r\angle\theta$).
3. Let $z_1 = 2\angle 180°$, $z_2 = \frac{1}{2}\angle 60°$ and $z_3 = 1\angle - 45°$.
    1. Plot z_1, z_2 and z_3 on an Argand diagram,
    2. Express $\frac{z_1 z_2}{z_3}$ in *both* Cartesian and polar coordinates.
4. Let $z = \frac{3}{\sqrt{2}} + \frac{3}{\sqrt{2}}i$. Show that the polar coordinates of z are $3\angle 45°$. Hence find $\sqrt{z}$. (*Both* square roots are required.)

Question Five

1.
   1. Evaluate any *two* of the following limits:
      1. $\lim_{x\to\infty} \frac{7x^5 - 3x^3 + 3x - 6}{x^5 + x^4 + x^3 + x^2 + x + 1}$
      2. $\lim_{x\to 2} \frac{x^2 - 4x - 12}{x^2 - 7x + 6}$
      3. $\lim_{x\to 1} \frac{e^{3x-1} - 1}{x - 1}$ (*hint:* L'Hôpital's Rule)
   2. Find $f'(x)$ where:
      1. $f(x) = (x^2 - 7x + 4)\tan(x^2 + 1)$
      2. $f(x) = \frac{\sin\left(x^2+3x+2\right)}{x^2+3x+2}$
   3. Find $\frac{dy}{dx}$ where $xy^2 - 2xy - x^2 + y^2 + 1 = 0$, and hence find the gradient of the tangent to this curve at the point $(1, 1)$.
   4. Use logarithmic differentiation to find $\dfrac{dy}{dx}$ where

$$y = \frac{(x^2 + 7)^5 (x^2 + x + 1)^{12}}{2x^3 - 6}$$

Question Six

1. Find the turning points of the graph of the function $y = 2x^3 - 3x^2 - 12x + 12$. Distinguish between them and sketch a graph of the function.
2. A farmer wishes to fence of $2000m^2$ of land adjacent to a road. It costs 30 euro per metre to fence off the land adjoining the road, but only 20 euro per metre to fence off land not adjacent to the road. It is also required to divide the fenced off area into three by erecting two electric fences, the same length as the field, perpendicular to the road, at a cost of 10 euro per metre. Assuming that the area to be fenced is rectangular:
    1. How long should the length of fence along the road be if the cost of the fencing is to be minimised,
    2. what will the total cost of the fencing be?

Question Seven

1. Answer any *three* of the four parts of this question:
   1. Find $\int (y^2+1)\sin(y^3+3y-1)dy$
   2. Evaluate: $\int_0^1 xe^x dx$
   3. Find $\int \frac{x+4}{x^2+5x+6}dx$
   4. Determine the area enclosed by the curve $y = 4 - x^2$ and the x-axis.

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- About 20% of people that do apply and are offered courses do not take them up.
- A substantial number of people do the course purely for their personal development.

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- Few graduates from the foundation course who start degrees fail to finish
- Those who do drop out do so for non-academic reasons
- Most do science degrees, followed by engineering. A few people went on to do maths degrees.

Some Do Very Well

Figure: Dr. Salvatore Polizzi McDonagh Ph.D in Biomedical Engineering

Some Do Very Well

Figure: Caroline Martin, First Class Honours in Earth and Ocean Science, went on to do a PhD in Cambridge

Some Do Very Well

Figure: Anthony Perkin B.E, First class honours, and top of the class every year, in Civil Engineering

- Don't forget, that for many of the foundation course graduates, obtaining any sort of a degree was the stuff of their dreams.

Figure: Some graduates

How Does Teaching Mature Students Differ?

- They have a fear of mathematics
- They tend to look at what they can't do rather than what they can
- They ask questions!! (Lots of them)
- They expect you know everything!

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- They expect you know everything! *keeps you on your toes*

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- Skemp was a psychologist, so he would say that, but I believe that by-and-large it is true.

From "How To Solve It"

Figure: George Polya

The Teacher of mathematics has a great opportunity. If the allotted time is filled with drilling his students in routine operations he kills their interest, hampers their intellectual development.

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But if the teacher challenges the curiosity of the students by setting them problems proportionate to their knowledge, and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of, independent thinking. But if the teacher challenges the curiosity of the students by setting them problems **proportionate** to their knowledge, and helps them to solve their problems with *stimulating* questions, he may give them a taste for, and some means of, independent thinking.

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- Whilst both the above quotations are true for *all* students, they are especially true when it comes to mature students.
- It is also true that as mature students are not stepping straight out of second level education they are in general less hampered by a belief that "maths is something that is drilled into you". So this presents opportunities.

So what do you (I) do?

- You encourage, encourage, encourage . . .
- The "lectures" are really a mixture of lectures and tutorials—lecture in short bursts, give them exercises to try in between.
- Give a range of exercises of varying difficulty, but point out which ones are difficult and do all you can to make people that you are having problems with the more difficult problems believe that they are capable of solving them with *practice*

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- *Dispel the myth that people are either born good at mathematics, in which case they can do it by magic, or bad at it, in which case it is pointless trying.*

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- This encourages a belief that there is some point to differentiation, and that it might be something that is worth putting a bit of effort into.
- Then you can go back to differentiation from first principals, limits, product rules etc.

How is this done

- Start by stating a rough definition of differentiation as the study of how fast functions are changing at a given point.
- Show diagrammatically that if $y = mx + c$ then $\frac{dy}{dx} = m$
- Next state informally that the slope of a curve at a given point is the slope of the tangent to the curve at that point.
- Ask them to accept that $\frac{d}{dx}x^2 = 2x$, backing this up with some diagrammatic evidence, and stating that we will return to why $\frac{d}{dx}x^2 = 2x$ later.
- Explain why it makes sense that a function has a turning point where $\frac{dy}{dx} = 0$
- Now find the turning point of a quadratic function

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- Thus $y = \frac{100}{x^2}$ so $S = x^2 + 4x\frac{100}{x^2} = x^2 + \frac{400}{x}$
- Ask them to accept for the moment that $\frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2}$, use a couple of diagrams to show this makes sense.
- and solve the problem.

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- I was pleased with the level we got the students to in a short space of time
- I spent more time at "nuts and bolts" than I would like
- I did take side steps from the course occasionally and show that $\sqrt{2}$ is irrational, that there is no greatest prime number, etc. Many students do appreciate this.
- If you asked former students how they feel the course should be changed, they invariably were happy with the way it was

That's It!!

- Thanks for Listening
- Any Questions?