Fine tuning traditional and innovative teaching to enhance learning

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Outline

Context

Rationale and aims

Implementation: methods and student response

Discussion

The question

Is there a teaching method/technique that leads to best learning?

Not generally, as it may well depend on

1. Specific content and type of learning;

2. Context;

3. Learner’s individual preferences.

Context: my half of the Unit

Maths for Scientists 4

-A Year 2 Semester 2 Unit

Context: my half of the Unit

Ten lectures and 3 problem classes (3 problem sheets)

Assessment: 100% exam.

A definition: the “problem class”, or “tutorial”

Definition

Problem class: learning environment where questions constructively aligned to Unit’s Intended Learning Outcomes (ILOs) can be informally practiced.

E.g.:

1. (a) Define a linear operator;

   (b) Hence prove whether the following operator L is linear or not:

$$L: f(x) \mapsto 2\frac{d^2}{dx^2}f(x) + 3xf(x) - \frac{4}{x}.$$

Context: my half of the Unit

Maths for Scientists 4

- main topics covered in my half of the Unit:

- **Linearity** of operators;
- Partial Differential Equations (primarily diffusion, wave, Laplace, Schrödinger) solved by **separation of variables method**;
- Convergence of **power series**;
- **Power Series Methods** applied to Ordinary Differential Equations (Legendre, Hermite, Bessel).

Context: educational

- Experiment new methods of teaching and learning;

- Social constructivist approach (Vygotskian *“zone of proximal development”* and *“more knowledgeable other”*);

- Create an environment that favours:

    - social interaction and exchange with peers;
    - questioning, discussing, challenging;
    - freedom of making mistakes towards the construction of own individual knowledge.

Aims

Promote ***deep learning*** through:

1. Implementation of a range of learning activities that involve social interaction;

2. Establishment of an ongoing dialogue to allow two-way - individual and group – feedback.

Aims

Evaluate effectiveness through:

1. Analysis of exam marks in various exam questions.

2. Student participation and feedback on the various methods.

Implementation

The range of learning methods

Implementation: e-tools

Electronic Voting System (EVS),
aka “**clickers**” - Turning Point™;

Mobile Digital Visualiser,
aka “**digital pen**” - Papershow™.

Range of learning methods

Traditional “chalk and talk”;

Enhanced “chalk and talk” (handouts with gaps to be filled in);

The inverted lecture;

Digital pen to work out solutions in lectures (interaction-enhanced lectures);

Traditional problem classes;

Clicker and digital pen sessions.

Range of learning methods

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Delivery of material

Handouts with gaps to be filled in (“**enhanced *chalk and talk***”);

The inverted lecture.

Delivery of material

Handouts with gaps to be filled in ("enhanced *chalk and talk*");

The inverted lecture.

Handouts with gaps

Lecture 3

2.2

Theorem

Consider the general equation

$$\mathcal{L}\phi = S(\vec{r},t) \qquad (2.2)$$

with *L* linear operator. If a function *ψ* satisfies

$$\mathcal{L}\psi = 0\,, \qquad (2.3)$$

then φ+*aψ*, *a* arbitrary constant, is also a solution of (2.2).

Handouts with gaps

Proof

$$
\begin{aligned}
\mathcal{L}(\phi + a\psi) &= \mathcal{L}(\phi) + a\mathcal{L}(\psi) && \text{because } \mathcal{L} \text{ is linear} \\
&= S + 0 && \text{because of (2.3)} \\
&= S,
\end{aligned}
$$

i.e. $\varphi + a\psi$ satisfies (2.2). [QED]

In general, (2.3) can have many solutions, often an infinite number; write these as ψ_α. The solution to (2.2) can then be written as

Handouts with gaps

$$\phi + \sum_{\alpha} A_\alpha \psi_\alpha \qquad (2.4)$$

where A_α are constant coefficients. (2.4) is called the **GENERAL SOLUTION**.

To obtain a specific solution from the general solution (2.4), we need to use **BOUNDARY CONDITIONS** to fix the values of the coefficients A_α.

<u>EXAMPLE</u>

The wave equation in 1D with no source term (S = 0):

$$\frac{\partial^2 \phi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} = 0 \ .$$

Handouts with gaps

Two solutions are

$$\Phi(x,t) = e^{i(kx-kct)} \quad \text{and} \quad \Phi(x,t) = e^{i(kx+kct)}$$

for any value of *k*.
The general solution is then

$$\Phi_k(x,t) = \underbrace{\sum_k}_{\text{sum over all possible values of } k} \Big[\underbrace{A_k}_{} \cdot e^{i(kx-kct)} + \underbrace{B_k}_{} e^{i(kx+kct)} \Big] \qquad (2.5)$$

a pair of coefficients for each k.

Handouts with gaps

The values of A_k and B_k will be fixed by the boundary conditions specific to the problem. Suppose these are (at t = 0, also called *initial conditions*):

$$\Phi(x,0) = \sin(3x) \qquad (☼)$$

$$\frac{\partial \Phi(t=0)}{\partial t} = \dot{\Phi}(x,0) = 0 \qquad (☼☼)$$

Use (☼☼) first:

$$\dot{\Phi}(x,t) = \sum_k \left[-ikcA_k e^{i(kx-kct)} + ikcB_k e^{i(kx+kct)} \right]$$

$$\dot{\Phi}(x,0) = \sum_k \left[-ikcA_k + ikcB_k \right] \cdot e^{ikx}$$

$$\stackrel{!}{=} \sum_k -ikc(A_k - B_k) \cdot e^{ikx} = 0, \text{ for all } x.$$

Handouts with gaps

This last condition is true if and only if $A_k = B_k$, so (2.5) becomes

$$\phi(x,t) = \sum_k 2A_k e^{ikx} \cos(kct), \quad (2.6)$$

since $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$.

Delivery of material

Handouts with gaps to be filled in (“enhanced *chalk and talk*”);

Inverted lecture.

Inverted lecture

PH20020 – Part 2 – Lecture 6
Vibrations of a Drum

1. **The question** What note(s) do you get when you hit a drum?

2. **Setting up the equation** The vibrations of a drum are governed by the wave equation

$$\nabla^2\Phi = \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} \quad (1)$$

where Φ describes the amplitude of the vibration and c is the speed of the waves, which is determined by the tension and mass of the drum-skin.

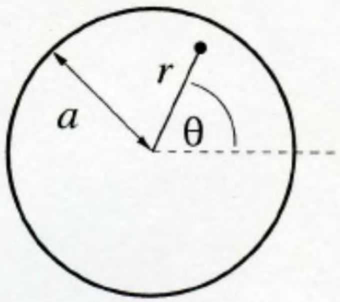
Consider the case of a circular drum. We then have in equation (1)

- ∇^2 must be two-dimensional, and
- it makes sense to use (cylindrical) polar coordinates.

The general wave equation (1) therefore becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2} = \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}. \quad (2)$$

The geometry we use is shown in the diagram below.



Inverted lecture

So, we want to solve equation (2) for $\Phi(r,\theta,t)$, where Φ represents the displacement of the drum-skin.

3. **Boundary conditions** There are two types of boundary condition in this problem. The first is governed by the physical fact that if the drum-skin is held rigidly around its perimeter, then the vibration amplitude must go to zero there. This can be expressed mathematically as

$$\Phi(a,\theta,t) = 0 \quad \text{for all } \theta \text{ and } t \tag{3}$$

where a is the radius of the drum.

The other type of boundary condition describes the way in which the vibrations are started. This will typically involve specifying the amplitude and speed of the drum-skin for all r and θ at some instant of time (ie, the *initial conditions*). However, to determine the frequencies of the drum (ie the note(s) it produces), it is not necessary to worry about the initial conditions, and we will leave them unspecified.

4. **Separation of variables 1** We first separate the spatial (r,θ) and time t variables, and look for a solution of the form

$$\Phi(r,\theta,t) = F(r,\theta)\,T(t). \tag{4}$$

Inverted lecture

How do you know which variables to separate first? Well, its a combination of trial and error, experience, physical intuition, etc, etc. In this case, I happen to know that separating (r,θ) and t first is going to give the right answer!

We substitute equation (4) into equation (2) to give

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial F}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 F}{\partial \theta^2}\right]T=\frac{1}{c^2}\frac{d^2T}{dt^2}F$$

or

$$\frac{1}{F}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial F}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 F}{\partial \theta^2}\right]=\frac{1}{c^2}\frac{1}{T}\frac{d^2T}{dt^2}=-k^2 \tag{5}$$

where $-k^2$ is the separation constant. As usual, it is important to choose the sign of the separation constant with care. In this case we want to have sinusoidal solutions in the t variable (because vibrations are basically oscillatory in time) and so we have $-k^2$ in equation (5).

5. **Equation for $T(t)$** The equation in t in (5) is

$$\frac{d^2T}{dt^2}=-k^2c^2T$$

which has solutions

$$T(t)=A_k\sin(kct)+B_k\cos(kct) \tag{6}$$

where A_k and B_k are constants that will be fixed by the initial conditions, as described above. Note that these constants have a k label. This is because the value of k is at the moment arbitrary and there are infinitely many solutions of the form of equation (6), corresponding to different values of k.

Inverted lecture

The Inverted Class

1. Why must the Laplacian be 2D (and not, for example, 3D)?
2. Why should we use (cylindrical) polar coordinates?
3. What could the initial conditions, for example, look like (try a mathematical description)?
4. Why do we choose the first separation constant (in (5)) to be negative? What would the time dependence of our solutions be like if we chose it to be positive?
5. Derive equation (9) from equation (7).
6. Why do we choose the second separation constant (in (9)) to be positive? What would the angular dependence of our solutions be if we chose it negative?
7. Why should Θ(□) be 2π-periodic?
8. Why does point 7. just discussed imply *n* is an integer?
9. Derive equation (12) from equation (11).
10. Why cannot Y_n be solutions of our problem?

Practice of material

Traditional problem classes;

Interactive clicker and digital pen sessions.

Practice of material

Traditional problem classes;

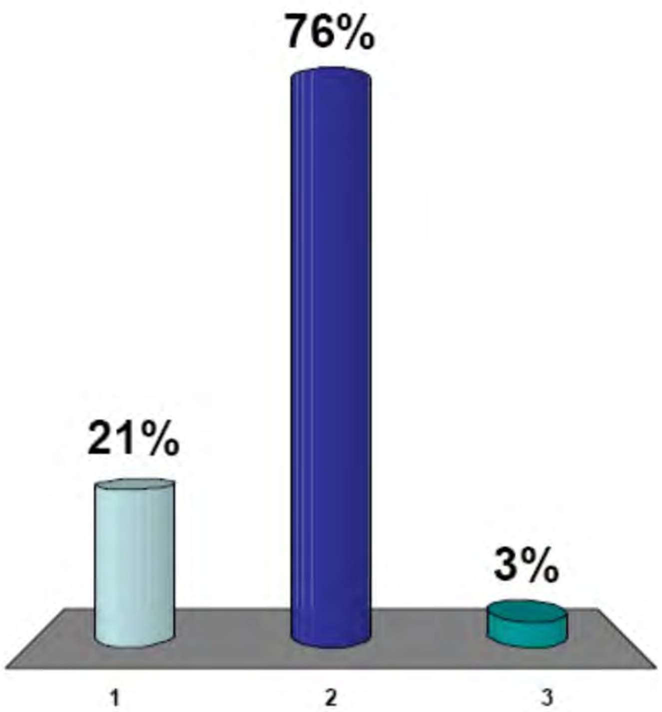
Interactive clicker and digital pen sessions.

Interactive clicker sessions

In the variables x and y, the function 1

$f(r,t) = e^{x-y}(x+xy)^{-1/2}$ is

1. Separable
2. Non-separable
3. Do not know

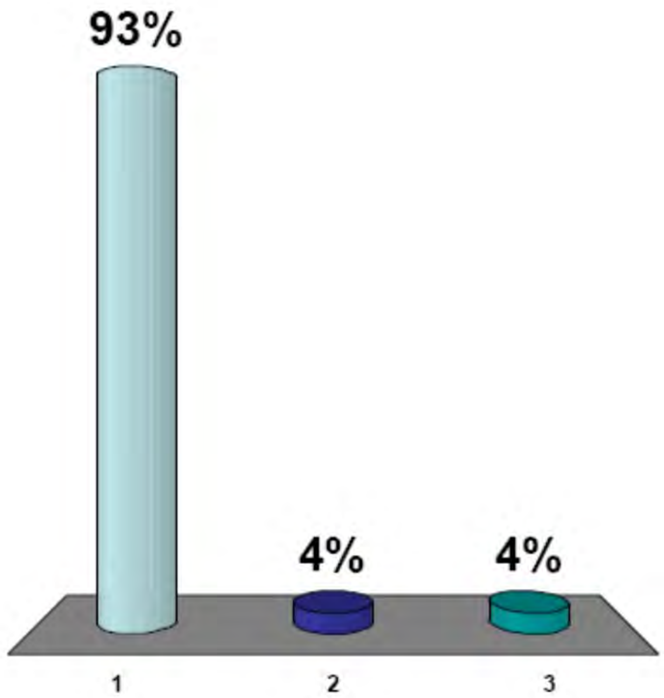


Interactive clicker sessions

In the variables x and y, the function 2

$f(r,t) = e^{x-y}(x+xy)^{-1/2}$ is

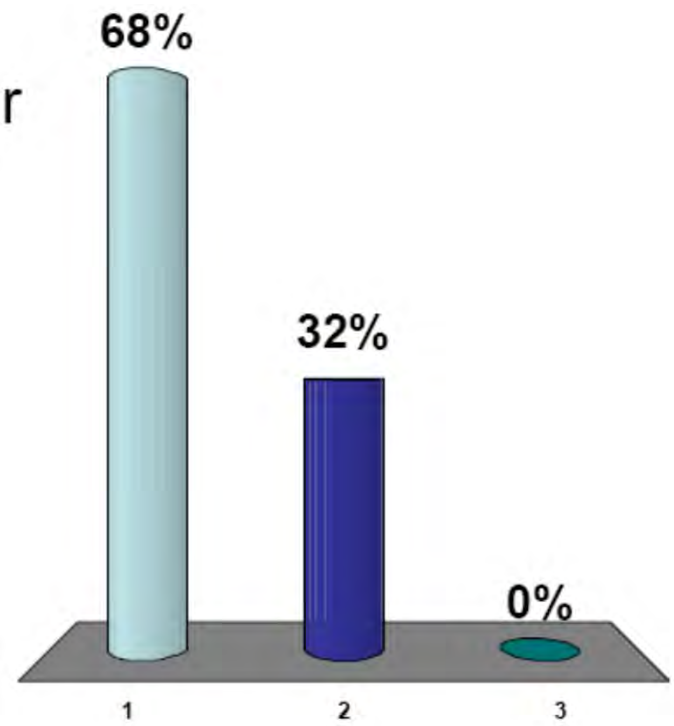
1. Separable
2. Non-separable
3. Do not know



Interactive clicker sessions

$\mathcal{L}$: f(x) → f(x)/x is (1)

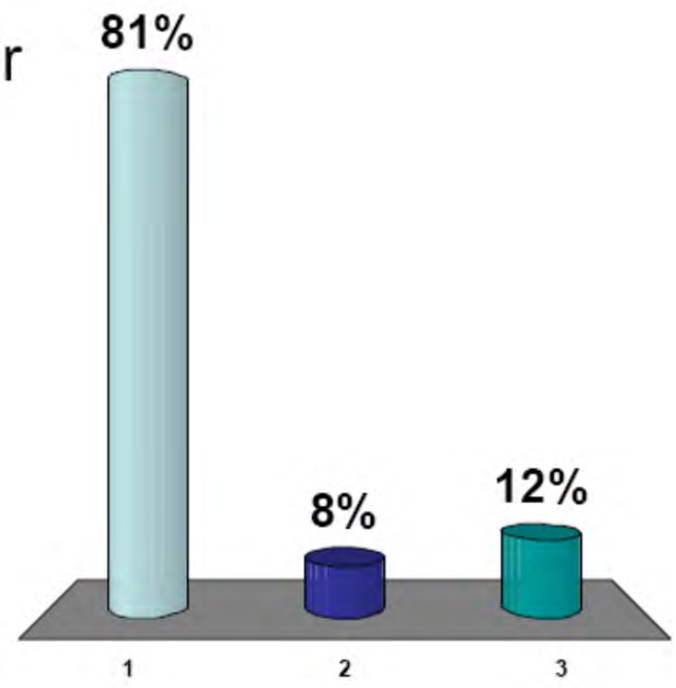
1. A linear operator
2. Not a linear operator
3. Do not know



Interactive clicker sessions

$\mathcal{L}$: f(x) → f(x)/x is (2)

1. A linear operator
2. Not a linear operator
3. Do not know



Interactive use of digital pen

$e^{x-y}(x+xy)^{-1/2}$

$$f(x,y) = e^{x-y}(x+xy)^{-1/2}$$

$$= e^{x}(e^{-y})(x)^{-1/2}(1+y)^{-1/2}$$

$$= \underbrace{(e^{x}x^{-1/2})}_{X}\underbrace{(e^{-y}(1+y)^{-1/2})}_{Y}$$

$$L(af+bg) = aL(f) + bL(g)$$

LHS: $\frac{1}{x}(af+bg) = \frac{af}{x} + \frac{bg}{x}$

RHS: $a\frac{1}{x}(f) + b\frac{1}{x}(g) = \frac{af}{x} + \vdots$

LHS = RHS $\therefore$ Linear

Interactive use of digital pen

$$L: f(x) \Rightarrow x^3 \frac{d^2}{dx^2}\left(f(x)\right) - x$$

Does $L(a\psi + b\phi) = aL(\psi) + bL(\phi)$?

$$L(a\psi + b\phi) = x^3\left(a\frac{d^2\psi}{dx^2} + b\frac{d^2\phi}{dx^2}\right) - x$$

$$aL(\psi) + bL(\phi) = a\left(x^3\frac{d^2\psi}{dx^2} - x\right) + b\left(x^3\frac{d^2\phi}{dx^2} - x\right)$$

$$= ax^3\frac{d^2\psi}{dx^2} - ax + bx^3\frac{d^2\psi}{dx^2} - bx$$

$$= \cancel{a}x(-a-b) + x^3\left(a\frac{d^2\psi}{dx^2} + b\frac{d^2\phi}{dx^2}\right)$$

$-x \neq (-a-b)x$ $\therefore$ this is not a linear operator !!

$$L: f(x) \rightarrow x^3 \frac{d^2}{dx^2}[f(x)] - x f(x)$$

$$L(a\psi + b\phi) \rightarrow x^3\left(a\frac{d^2\psi}{dx^2} + b\frac{d^2\phi}{dx^2}\right) - x(a\psi + b\phi)$$

$$aL(\psi) + bL(\phi) \rightarrow a x^3 \frac{d^2\psi}{dx^2} - a x \psi$$

$$+ b x^3 \frac{d^2\phi}{dx^2} - b x \phi$$

$$= x^3\left(a\frac{d^2\psi}{dx^2} + b\frac{d^2\phi}{dx^2}\right) - x(a\psi + b\phi)$$

$$\therefore \; aL(\psi) + bL(\phi) = L(a\psi + b\phi)$$

$\therefore$ <u>linear</u>

Implementation

Outcomes:

exam

and

student feedback

Implementation

Outcomes:

exam

and

student feedback

Implementation

Outcomes:

exam

and

student feedback

Implementation: exam questions

Q7. On linearity of operators: taught using enhanced chalk&talk, an interactive clicker&pen session and a traditional problem class.

Q8. Solve a PDE using separation of variables: taught using enhanced chalk&talk, a traditional problem class and an inverted lecture.

Q9. Use the power series method to solve a linear differential equation: taught using enhanced chalk&talk, an interactive clicker&pen session and a traditional problem class.

Implementation: exam questions

Q7. On linearity of operators: taught using ***enhanced chalk&talk***, an **interactive clicker&pen session** and a ***traditional problem class***.

Q8. Solve a PDE using separation of variables: taught using ***enhanced chalk&talk***, a ***traditional problem class*** and an **inverted lecture**.

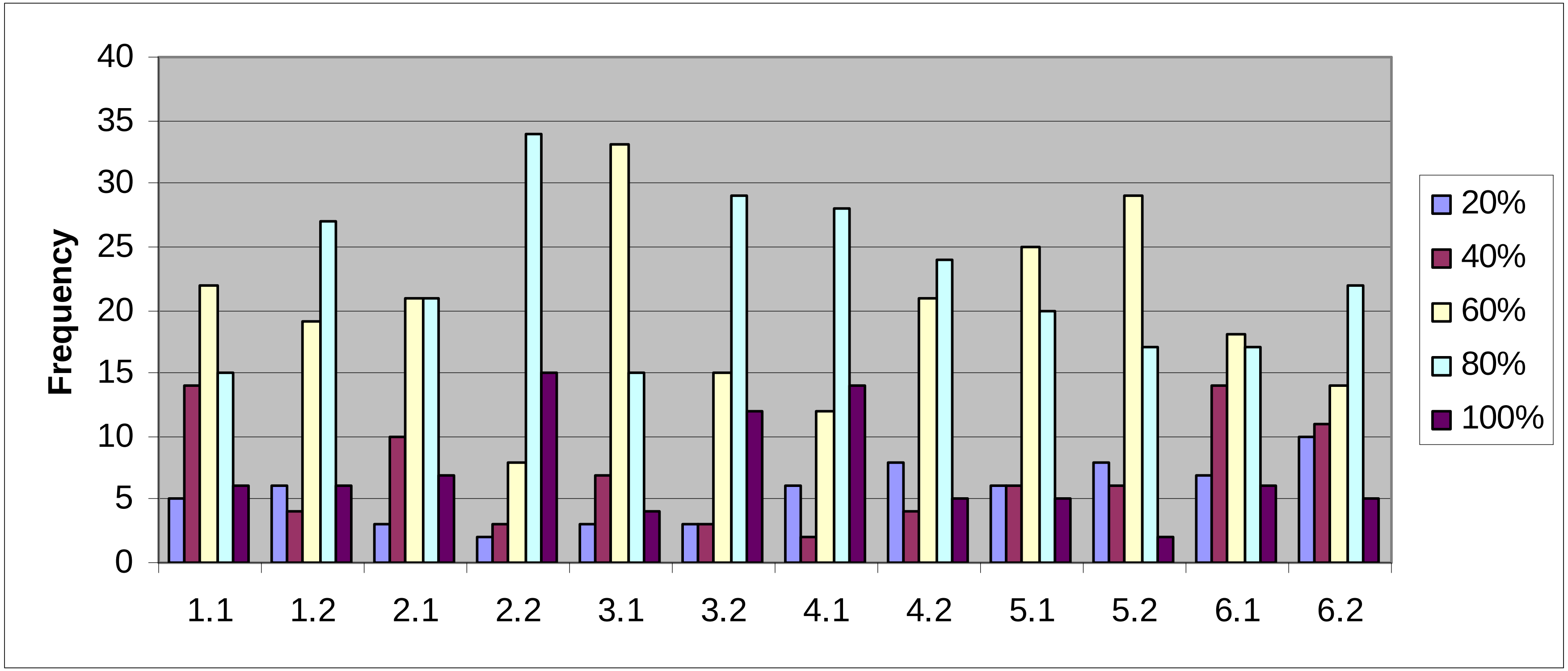
Q9. Use the power series method to solve a linear differential equation: taught using ***enhanced chalk&talk***, an **interactive clicker&pen session** and a ***traditional problem class***.

Implementation: exam results

Question	Mean	
7	54%	Interactive
8	60%	Inverted
9	61%	Interactive

Implementation: feedback

n.1 = enjoyable; n.2 = useful



Q1: traditional chalk and talk; Q2:hand-outs with gaps; Q3: traditional problem class; Q4: clicker sessions; Q5: digital pen; Q6: inverted class.

Implementation: feedback

n.1 = enjoyable; n.2 = useful

	Question	Mean	SD
Q1: chalk and talk	**1.1**	61%	23%
	1.2	67%	22%
Q2: hand-outs with gaps	**2.1**	66%	22%
	2.2	**78%**	19%
Q3: traditional problem class	**3.1**	62%	21%
	3.2	**73%**	23%
Q4: clickers	**4.1**	**73%**	26%
	4.2	63%	25%
Q5: digital pen	**5.1**	63%	23%
	5.2	**59%**	23%
Q6: inverted class	**6.1**	**59%**	25%
	6.2	**59%**	27%

Discussion: exam results

Lecture inversion

vs

interactive sessions:

There is no winner.

Discussion: student feedback

Tentative deductions:

From **student participation and feedback**:

Students join in in greater numbers than in the past, and a greater number contributes questions/answers in interactive sessions;

Students are used to questions in lectures being rhetorical!

Class can be fairly polarised with regards to preferences, but “enhanced chalk and talk”, i.e. **handouts with gaps, and traditional problem classes win over interactive sessions with clickers and digital pen and over inverted class** as far as **usefulness** is concerned, while **interactive sessions win** as far as **enjoyability** is concerned.

Discussion: limitations

1. Data analysis methodology: Likert scale (1-5) was handled by converting to % and looking at average values for each question: is this legitimate?

2. Experimenting with several methods over just 5 weeks can make students feel disoriented, perhaps even the subject of an experiment; this may lead to difficulty in establishing a relaxed, trusting and fruitful teacher-learner interaction. After all, the thing students enjoyed the most were the lecturer’s enthusiasm, excitement and passion.

Thank you for listening!