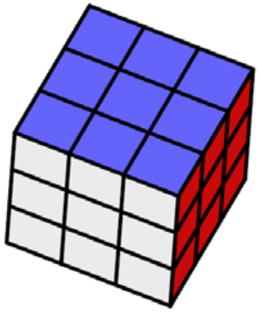
Magician's rope and Rubik's cubes:
Innovative Approaches to Teaching Pure Maths

Erik Baxter and Claire Cornock



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Maths at SHU

- Real, practical, current uses of maths
- Use of technology
- Development of employability skills

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However…

Pure modules

Year 1

- Number and Structure (Compulsory)

Year 2

- Linear and Discrete Mathematics (Compulsory)
- Introduction to Knot Theory (Elective)

Year 3

- Abstract Algebra (Elective)

Applications

This module aims to increase your understanding of how **abstract concepts can be used to form judgements about practical scenarios**.

This module aims to show you how Linear and Discrete mathematics is **applied in the fields of coding and cryptography.**

By engaging successfully with this module a student will be comfortable manipulating and analysing knots and links, **both in diagram form and using numerical / algebraic methods**

This module aims to develop your ability to understand abstract concepts and to **apply these ideas in a variety of different settings.**

Contextual Learning

‘Learning occurs only when students (learners) process new information or knowledge in such a way that it makes sense to them in **their frame of reference**…This approach to learning and teaching assumes that **the mind naturally seeks meaning in context**…and that it does so through **searching for relationships that make sense and appear useful.’**

Hull, 1993

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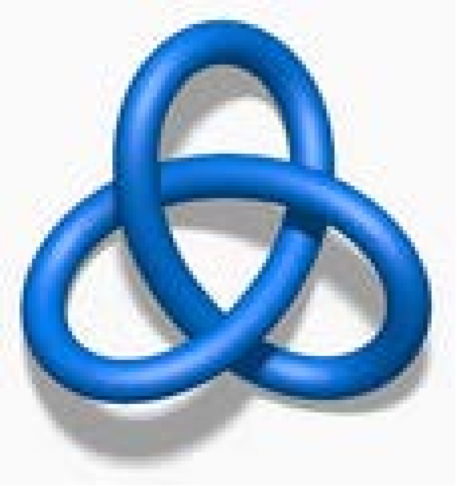
Examples

Topic	Aid	Modules
Knot theory	Rope	Introduction to Knot Theory
Group theory	Rubik’s cubes	Number and Structure Abstract Algebra
Monoid theory	Automata	Abstract Algebra
Permutations	Cards	Number and Structure

Rope



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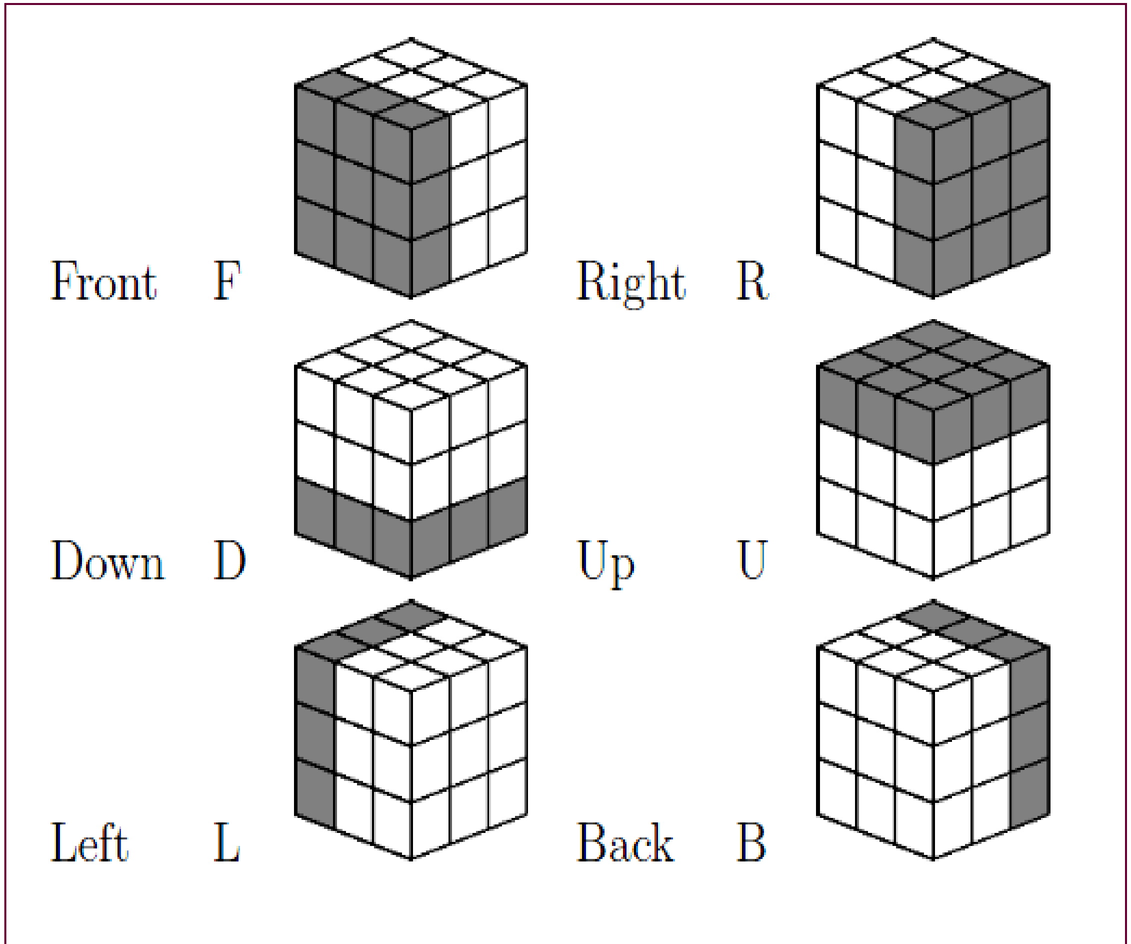
What the students think

"I was surprised by this lecture. I did not expect it to be as practical as it was. However this is a positive thing, as it **shows how maths is used with an application to something, rather than just theory on paper."**

"[The first knot theory lecture was] easily the most interesting module on the entire course. Although we pretty much just played with string with magnets in it, it was a **different and enjoyable way to be taught a complex subject** such as knots theory. I have previously looked at come of this kind of work but never really grasped it, however now... i feel like i **should be able to understand it quicker and easier**."

Rubik's cubes



F is moving the cube 90 degrees clockwise, etc

Topics:
Subgroups
Generators
Homomorphisms
Equivalence relations

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What the students think

"I liked the idea, and after using them for a while we could imagine the cube without actually having to do it."

"It's great fun and interesting to study. You spend a lot of time putting them back to the solved state."

"It's really interesting content. I have really enjoyed studying it."

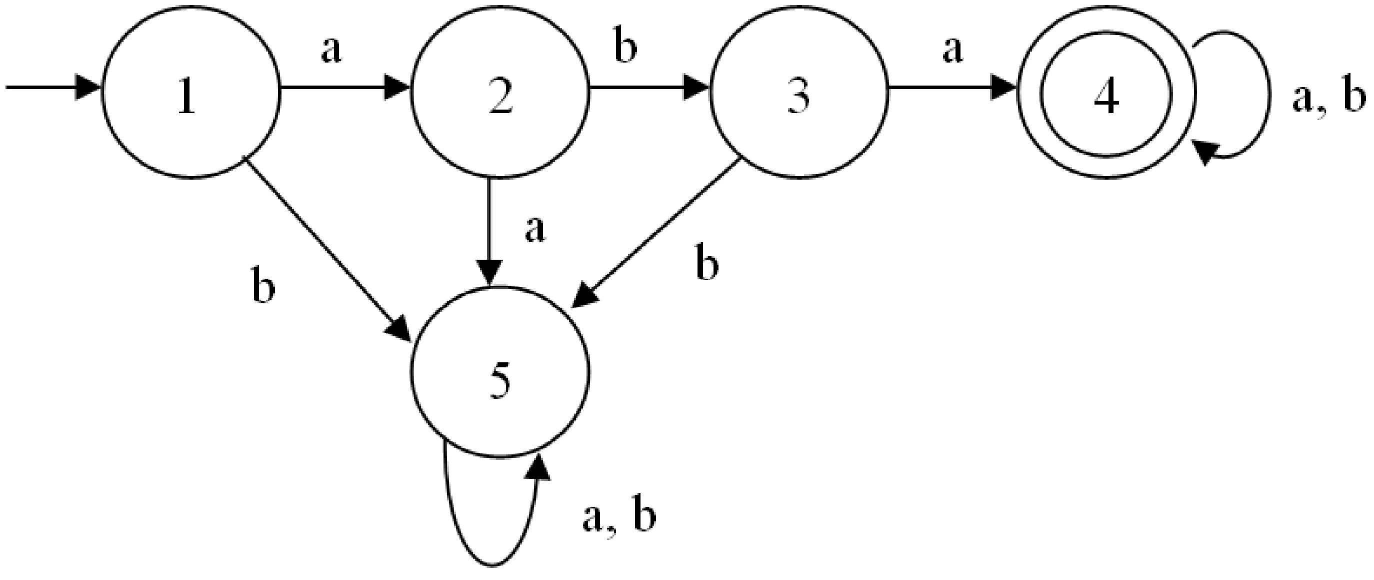
"Frustrating when I couldn't solve it."

"Hard, but interesting. Assignment was challenging."

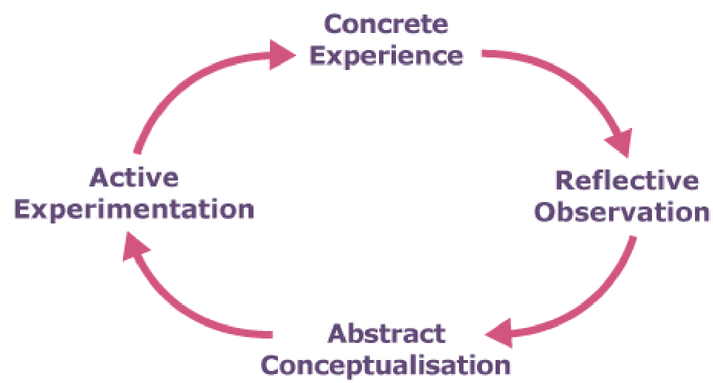
"Can't solve them if an error occurs."

Automata

A={a,b}



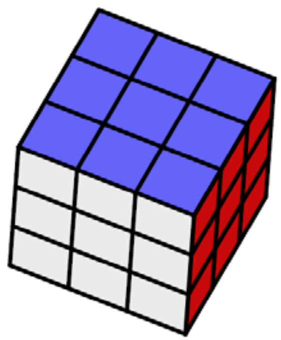
Teaching methods



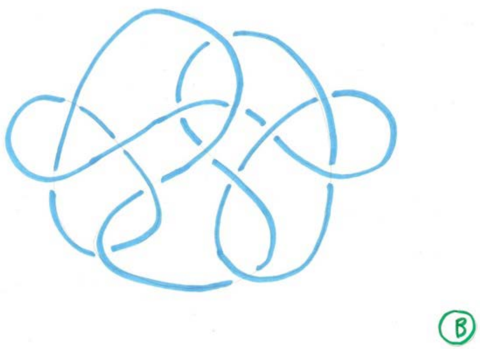
Kolb's learning cycle

'Active learning in small groups is much more like life after graduation than lecture learning is.'

Jackson, 1989



Assessment



Introduction to Knot Theory

50% exam

50% coursework (group task)

Abstract Algebra

50% exam

50% coursework (1 group task, 2 individual)

What the students think

"It's great to work with a group. The assignments are fun and interesting as well as **engaging**."

"Really enjoyed this - **learned a lot from one another."**

"This part really meant we **got to grips** with the semester 2 work."

"Enjoyed group work a lot and helped confirm a lot of subjects."

"A good way for the group members to **identify areas they need to work on**."

"The group assignment helped a lot for me, it was **great practice."**

"Was a good way to work and more **supportive**."

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Assessment

4) Investigate why $RU^{-1}R^{-1}U_m^2RUR^{-1}U_m^2$ only affects 3 pieces of a Rubik's cube. Write $RU^{-1}R^{-1}U_m^2RUR^{-1}U_m^2$ only using F, D, R, L, U and B. Explain each step.

(15 marks)

(b) **(i)** Explain what it means for a group G to be generated by a set X where X is a subset of G.

(ii) How many elements does $< R, L, R_m >$ contain? Why?

(iii) What are the possible orders for subgroups of $< R, L, R_m >$?

(iv) Give an example of a subgroup of each possible order. Justify why your examples have the orders that you claim they have. You do not need to provide multiplication tables.

(14 Marks)

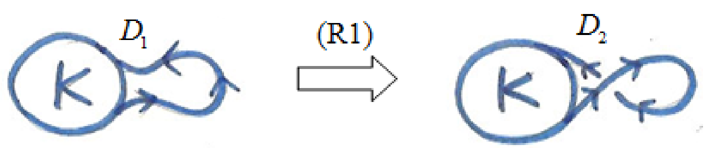
Assessment

5) Look at the knot tables. It seems that there are no knots with crossing number 1 or 2. Try and show why this is, by the following procedure:

a) Draw all 4 possible 1-crossing knots. This is as simple as drawing a single crossing and joining (and crossing) the strands in as many ways as you can. (Remember that knot diagrams related by planar isotopy are definitely equivalent, so you don't need to repeat yourself.) Satisfy yourself that they are all projections of the unknot.

b) Draw all possible 2-crossing *knot shadows*. (Adding the crossing information to make all possible 2-crossing knots will take too long.) Again, see if you can see why there are no non-trivial 2-crossing knots.

(b) Let D_1 and D_2 represent oriented diagrams of a knot K that differ by only one R1 move, in the following way:



(i) Explain why $\omega(D_2) = \omega(D_1) + 1$.

(2 Marks)

(ii) Show using the Kauffman bracket axioms (see formula sheet) that

$$\langle D_2 \rangle = -A^3 \langle D_1 \rangle .$$

(6 Marks)

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References

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