Self-Explanation Training in Undergraduate Mathematics

Self-Explanation Training for Mathematics Students

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Loughborough University

Students find it difficult to engage with proofs.

How do people read proofs?

Matthew Inglis →

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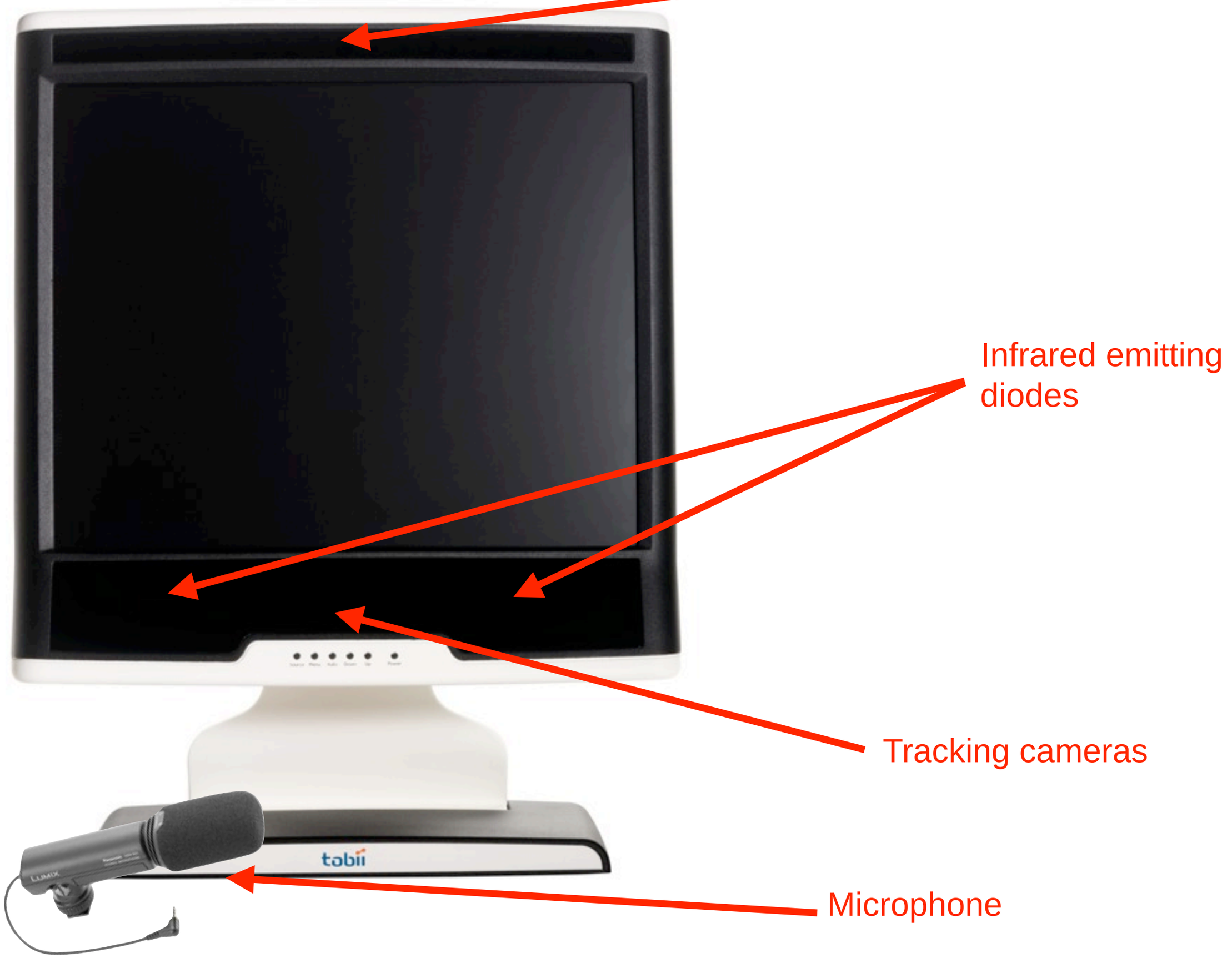
User camera
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During the first part of the experiment you will be asked to read a series of mathematical proofs, each written by a student in an examination.

Please read each proof and decide whether or not it is valid. When you are happy with your decision click the mouse button.

You should spend as long as you need reading each proof. Do not rush!

If you would like to speak as you read the proofs please feel free to do so.

If you get completely stuck, then click the mouse button to move on.

The first proof is for practice.

Click the mouse when you are ready to start.

Method

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Task: Read six proofs, decide whether each is valid or invalid, and judge confidence in answer.

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Analysis of focus: Total dwell time on formulae vs. non-formulae.

Analysis of reading order: Number of between-line saccades per proof.

Do students and mathematicians focus on different things?

Theorem. There are infinitely many primes that can be written as $4k + 1$ (where $k \in \mathbb{Z}$).

Proof. Suppose there are finitely many primes of the form $4k + 1$.

Then these primes can be listed $p_1,\ p_2,\ p_3, \ldots,\ p_n$.

Define a number a as follows. Let $a = p_1p_2p_3 \cdots p_n + 4$.

Note that dividing a by 4 leaves remainder 1.

Every number that leaves remainder 1 when divided by 4 is divisible by a prime that also leaves remainder 1 when divided by 4.

However, for all i such that $1 \leq i \leq n$, p_i divides $p_1p_2p_3 \cdots p_n$ and p_i does not divide 4.

Thus p_i does not divide a.

So dividing a by 4 leaves remainder 1 and a is not divisible by any prime that leaves remainder 1 when divided by 4.

This is a contradiction.

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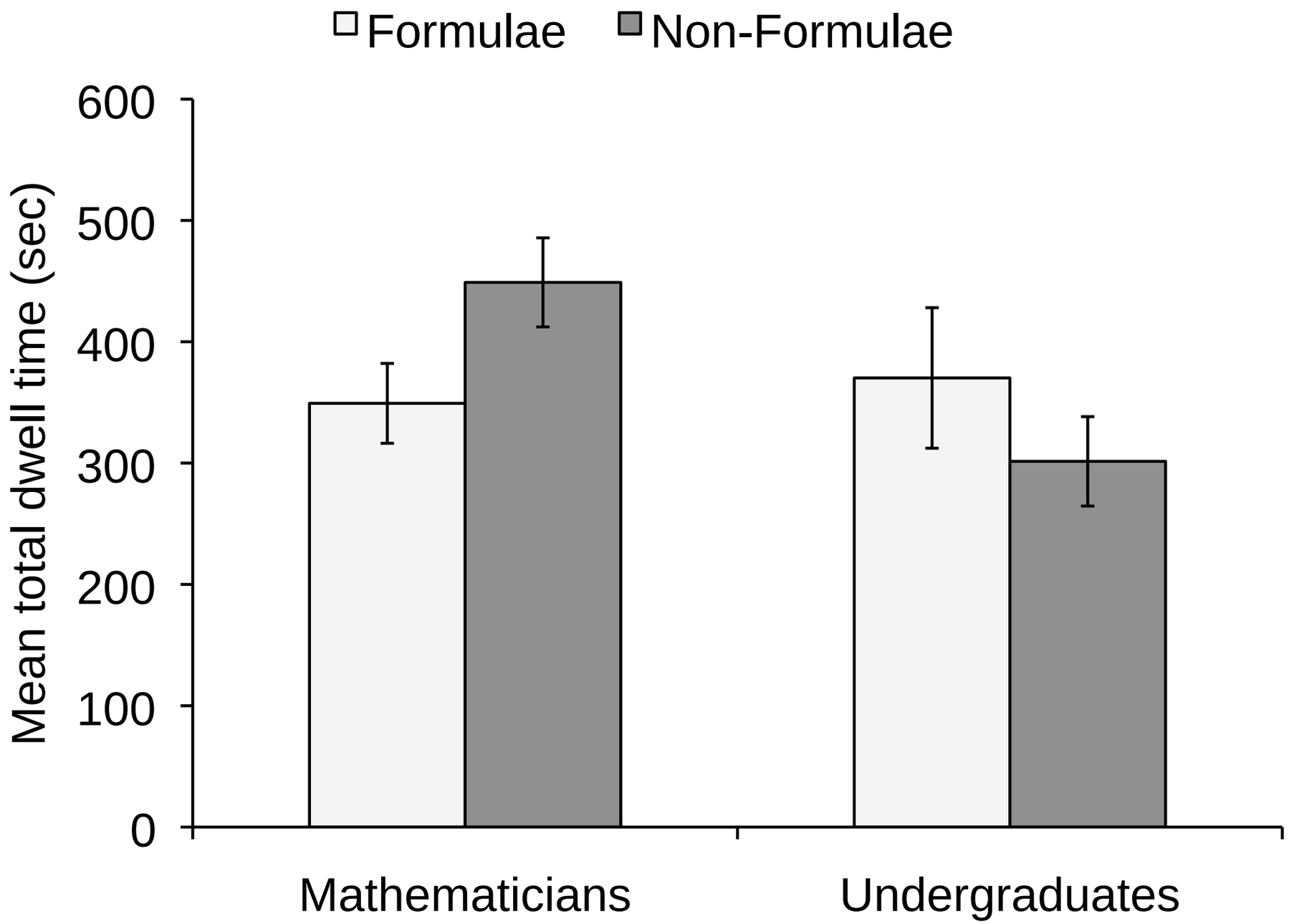

Focus on formulae

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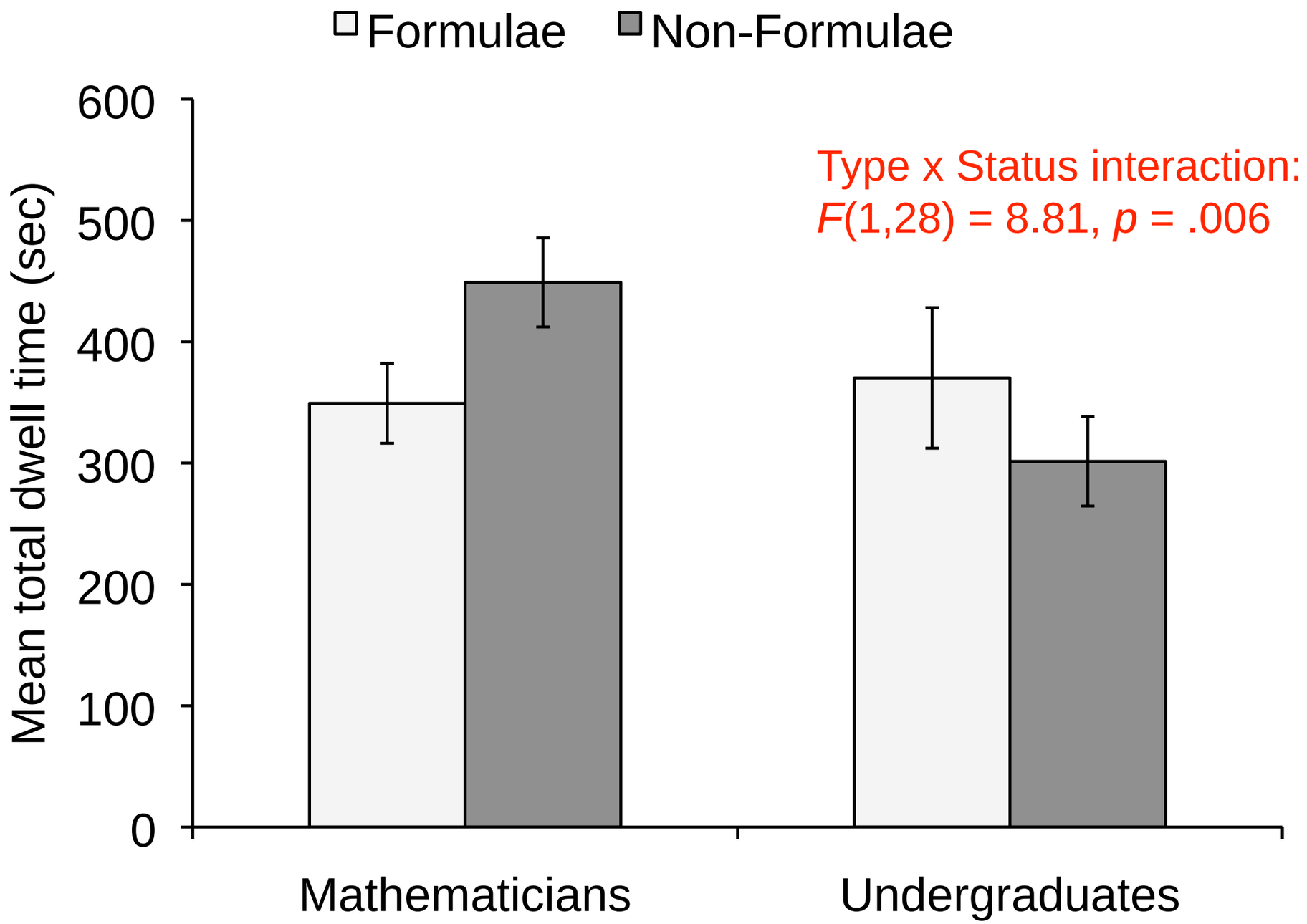
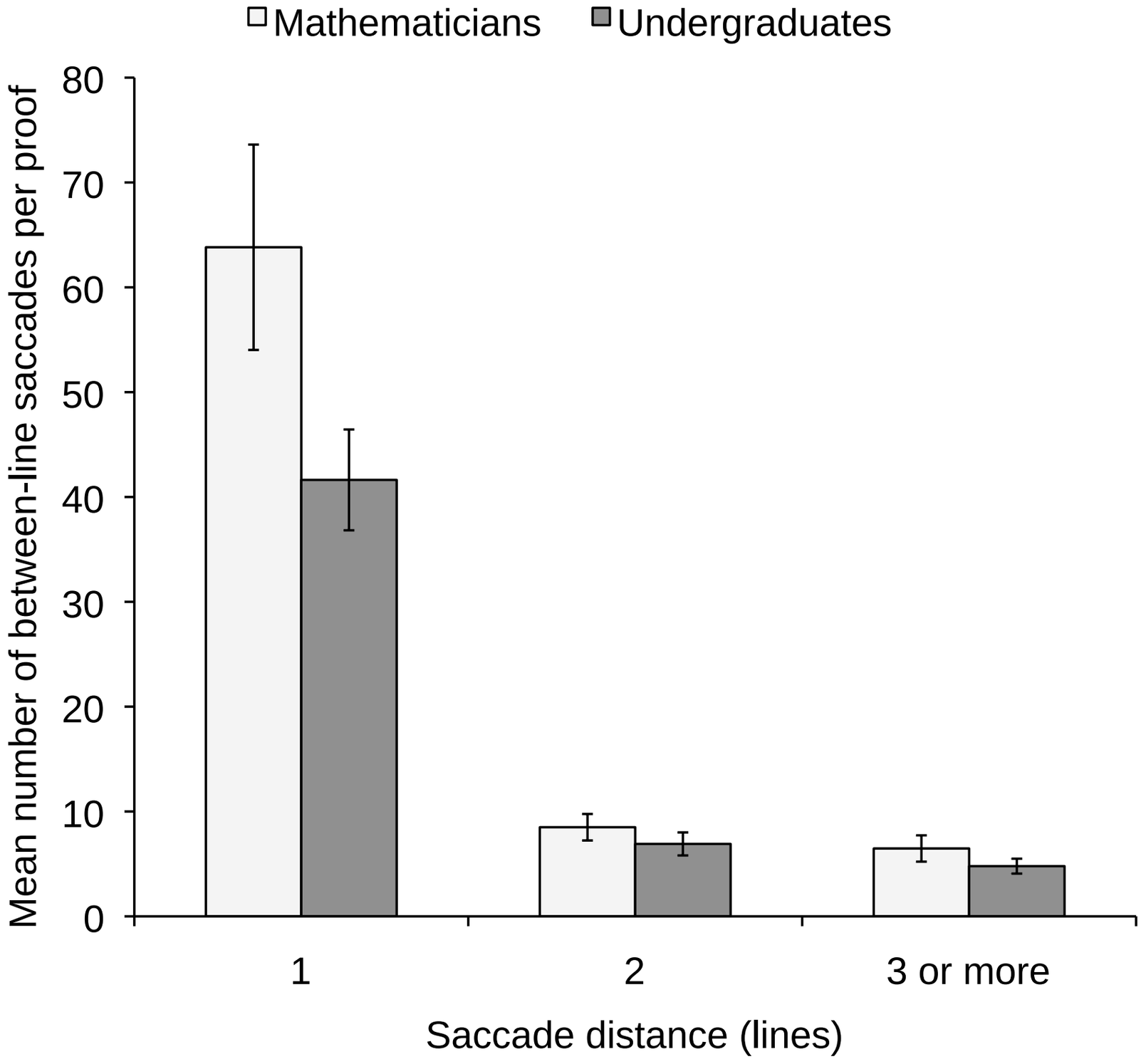


Type x Status interaction:
$F(1,28) = 8.81$, $p = .006$

Students focus proportionately more on formulae (less on the text).

Do students and mathematicians read in a different order?

Reading order



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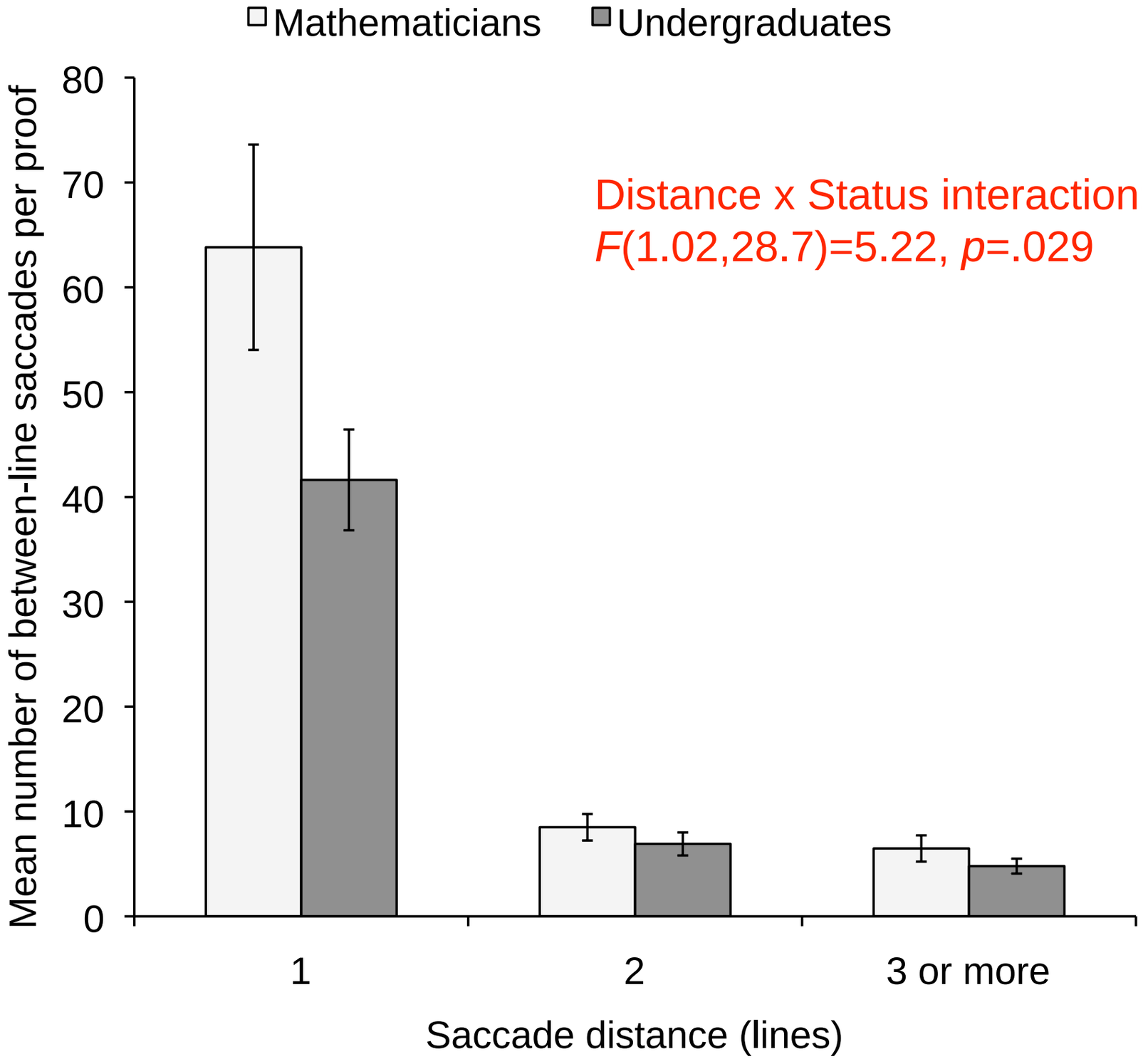

Distance x Status interaction:
$F(1.02,28.7)=5.22$, $p=.029$

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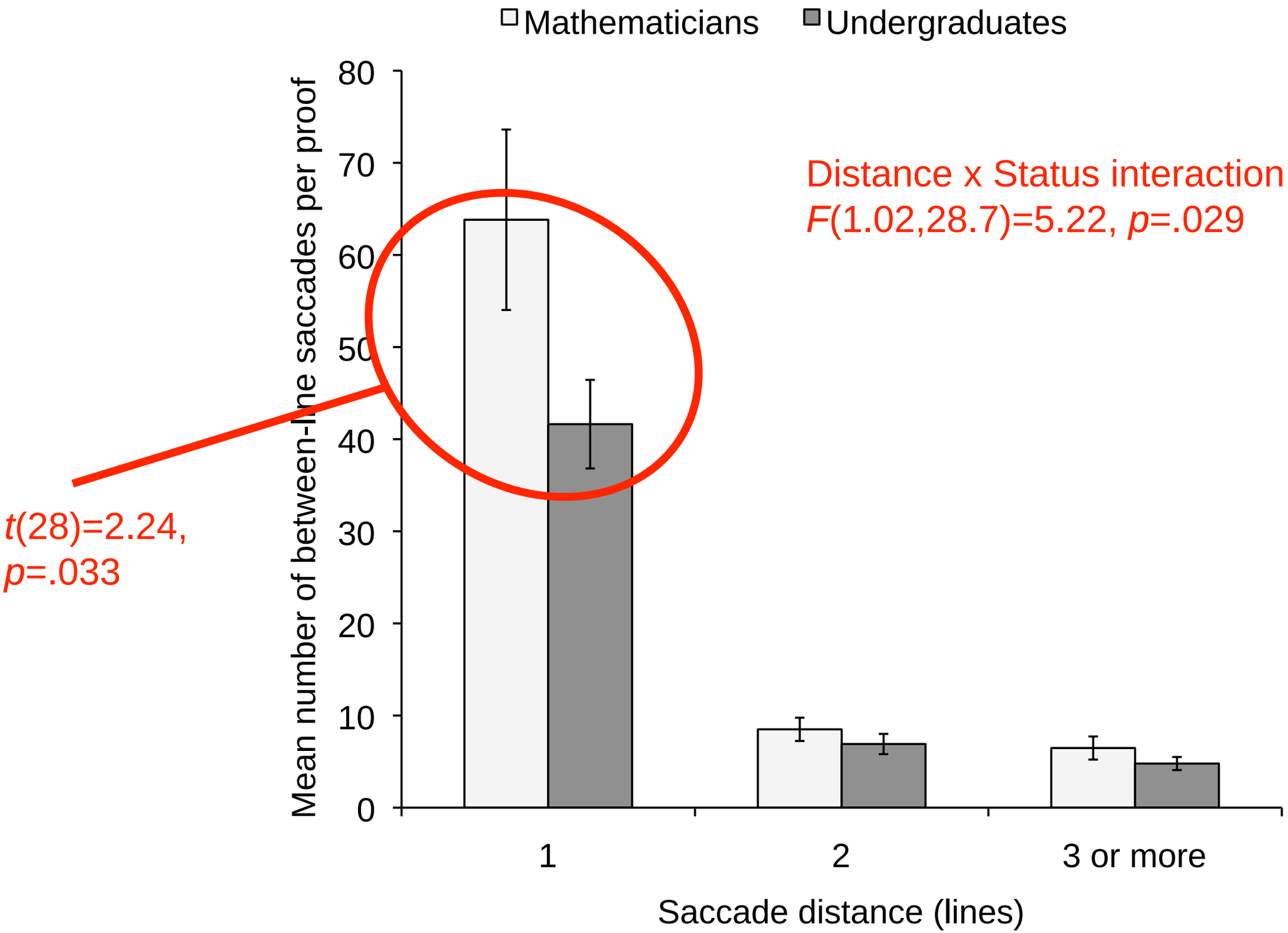

Distance x Status interaction:
F(1.02,28.7)=5.22, *p*=.029

t(28)=2.24,
p=.033

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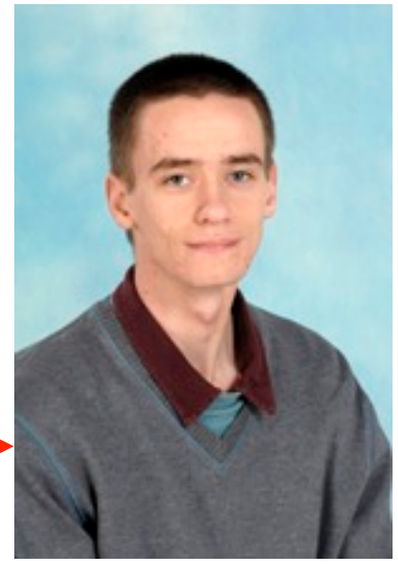
Mathematicians and students read differently.

Mathematicians move their attention around more.

Can we help students to read more effectively?

Mark Hodds



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Self-explanation training

Self-Explanation Training for Mathematics Students

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Self-study materials:

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Self-study materials:

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About 15-20 minutes.

Does self-explanation training improve comprehension?

Method

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Participants:

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Participants:

- 76 undergraduates.

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- Self-explanation and control groups (38 each).

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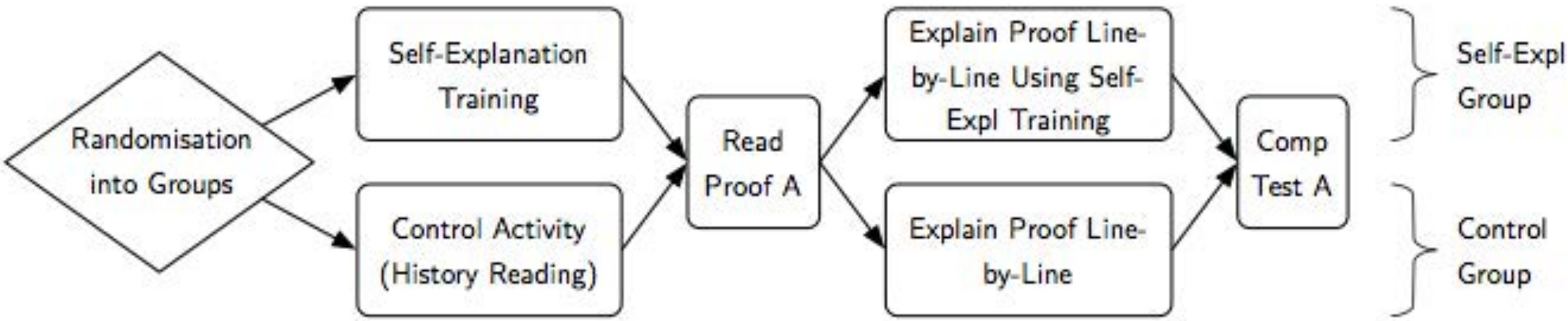

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Non-explanations:

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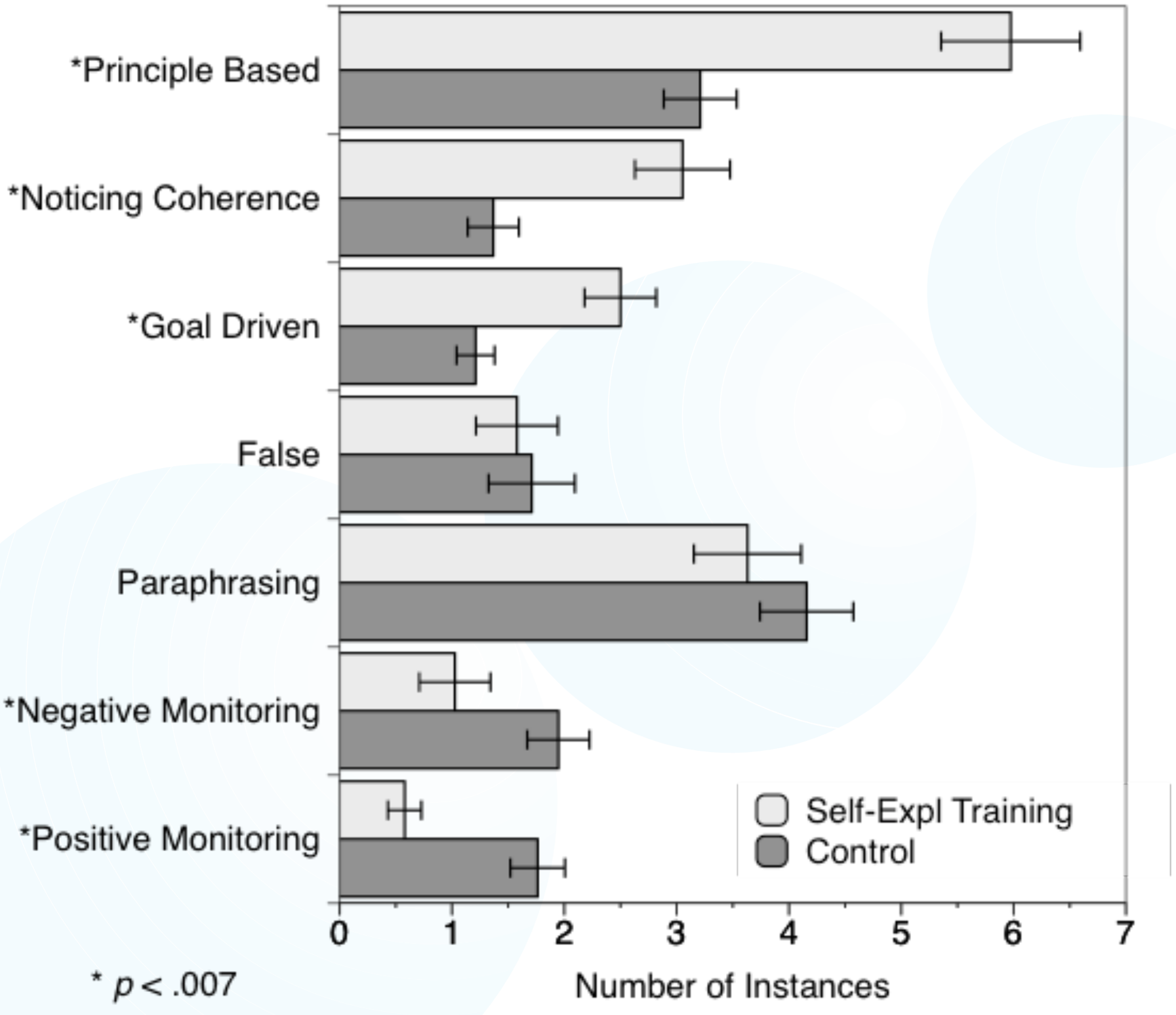
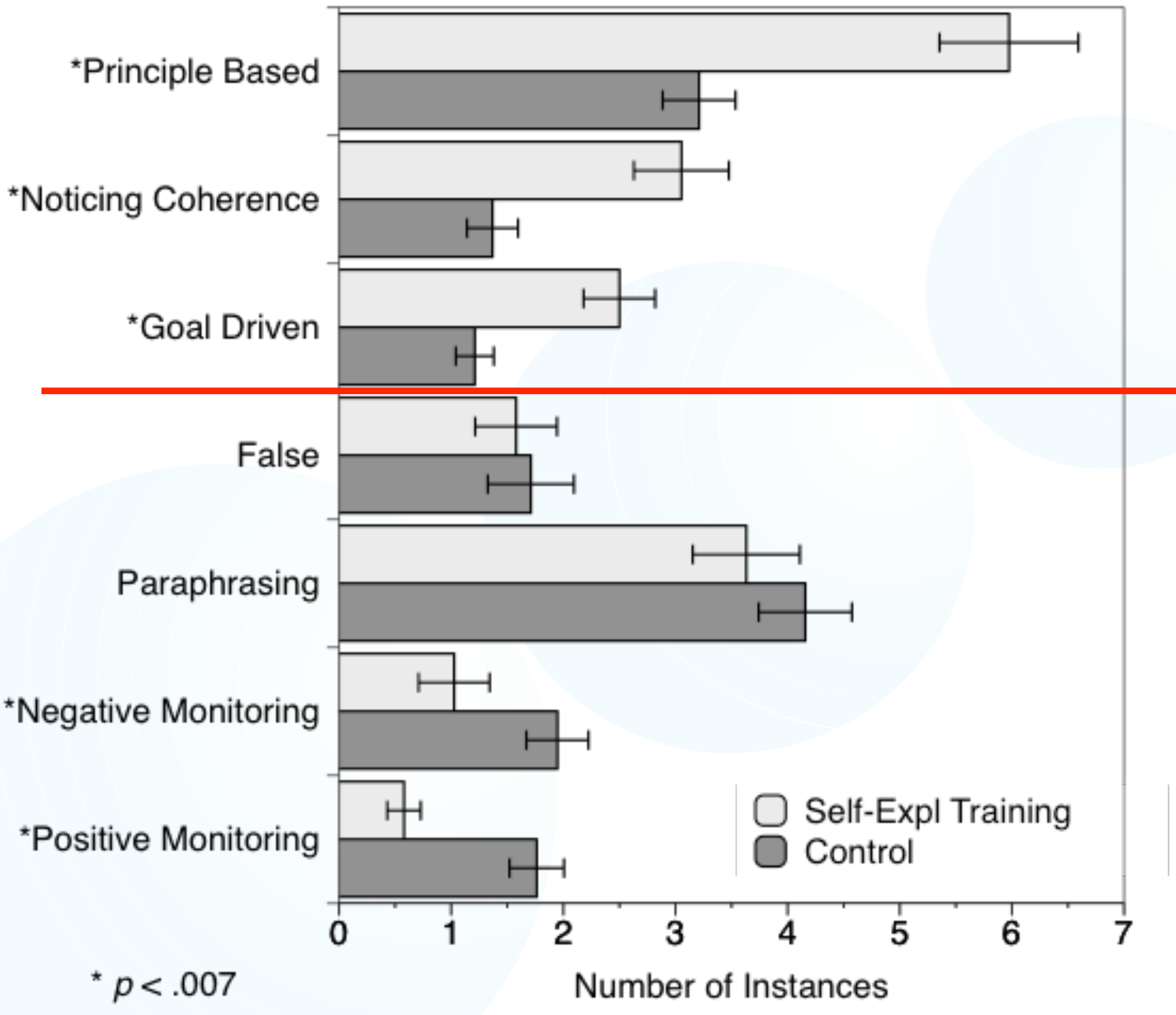
- Paraphrasing: repeat in similar or same words.
- Negative monitoring: “I don’t understand this”.
- Positive monitoring: “this makes sense”.

Explanation quality



* $p < .007$

Explanation quality



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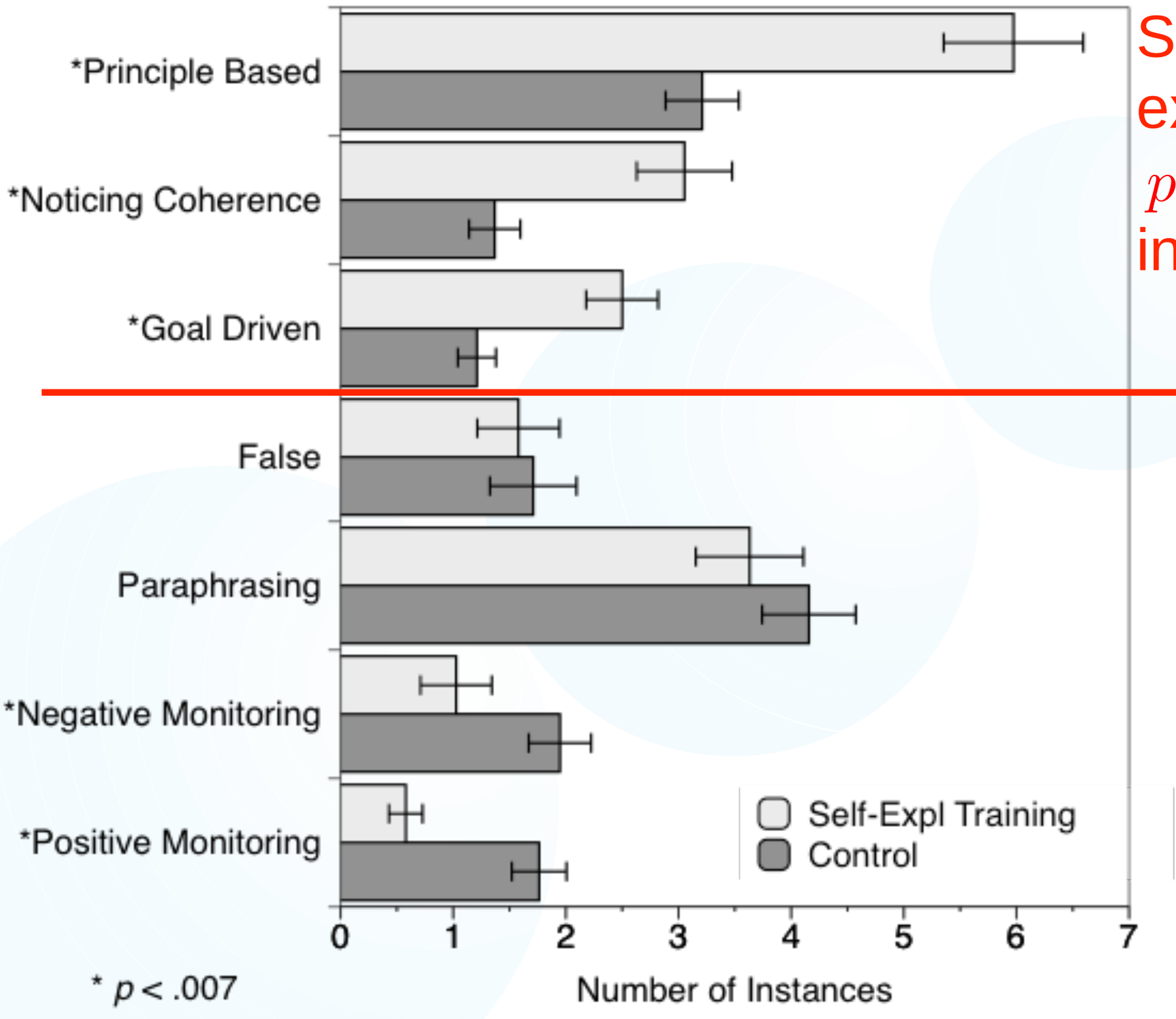


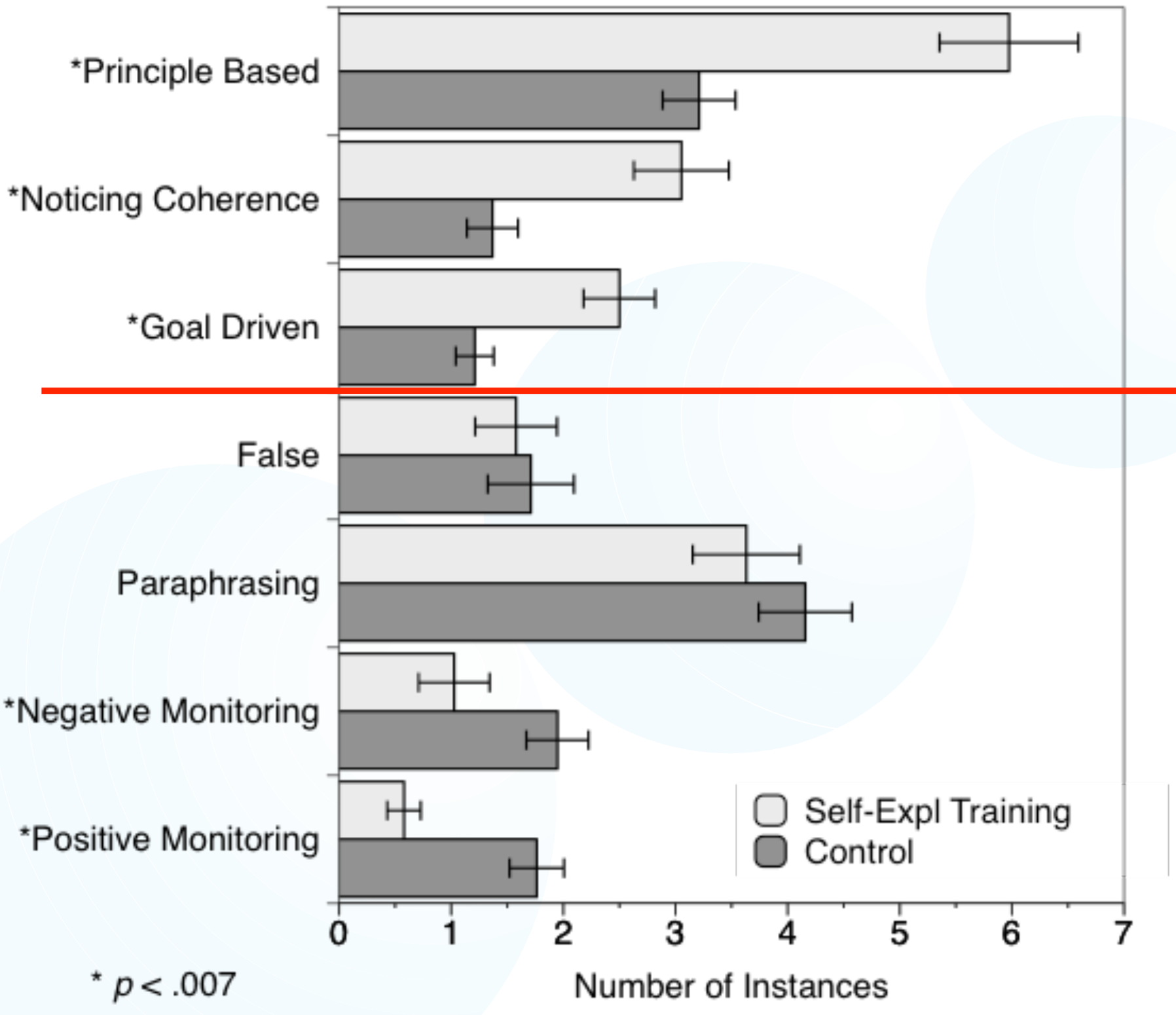
Significantly more explanations;

$p \leq .001$

in each case

* $p < .007$

Explanation quality



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Comprehension

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ANCOVA:

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- One between-subjects factor (condition: self-explanation, control).

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- Control group: 14.2 (SD=4.0)

Effect size: very large, *d*=0.950.

Self-explanation training leads to higher-quality explanations and better proof comprehension.

Does self-explanation change underlying reading behaviour?

Reading behaviour

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Approach: Eye-tracking with no requirement to think out loud.

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Participants: 32 undergraduates (data from 28 used).

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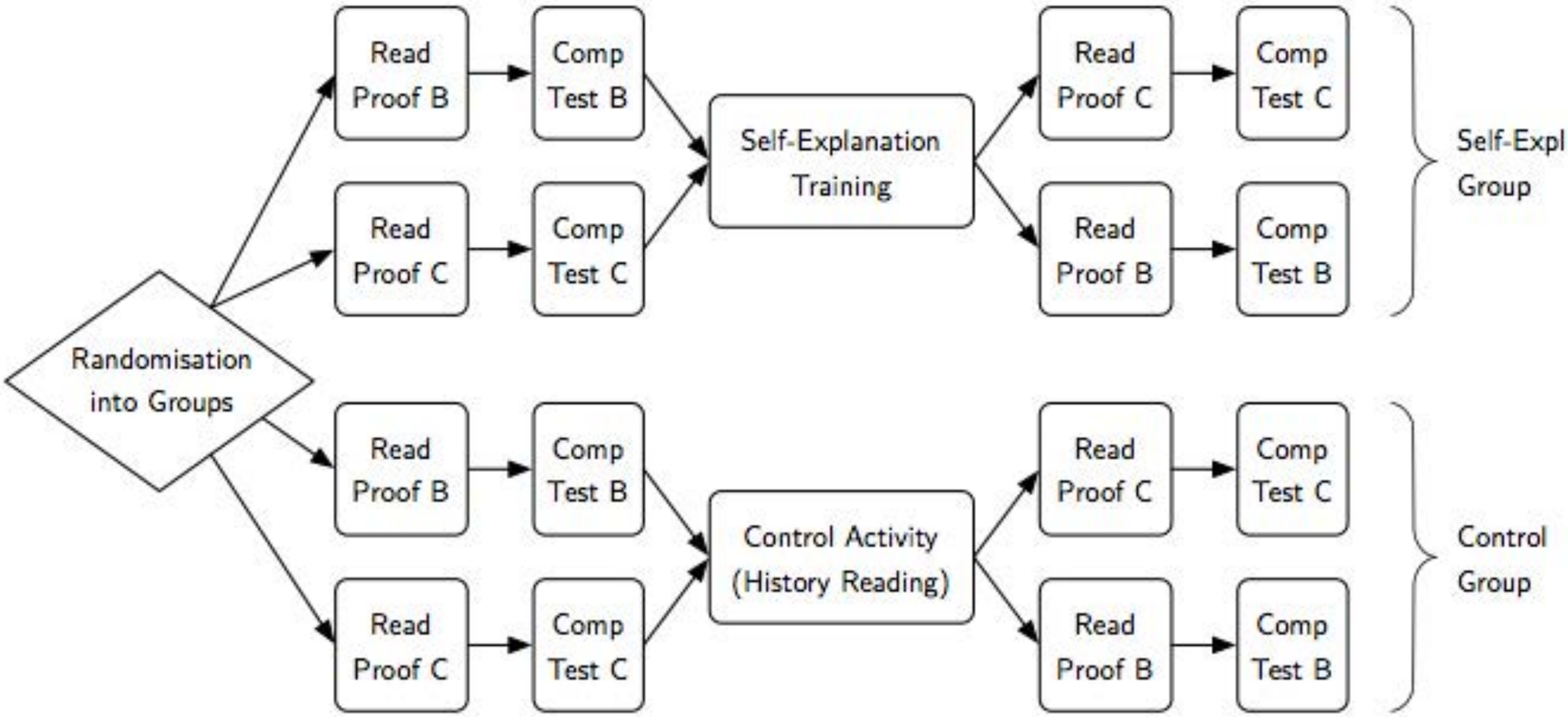

Cognitive effort

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Measure: mean fixation durations.

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$$F(1, 23) = 14.234, p = .001, \eta_p^2 = .382$$

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- Mean fixation durations for proof read first included as a covariate.

Significant effect of condition:

$$F(1, 23) = 14.234, p = .001, \eta_p^2 = .382$$

Average mean fixation durations on second proof:

- Self-explanation group: 301ms (SD=33.5)
- Control group: 276ms (SD=30.0)

Attention to logical relationships

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Measure: number of between-line transitions on proof read second (overall reading time as covariate).

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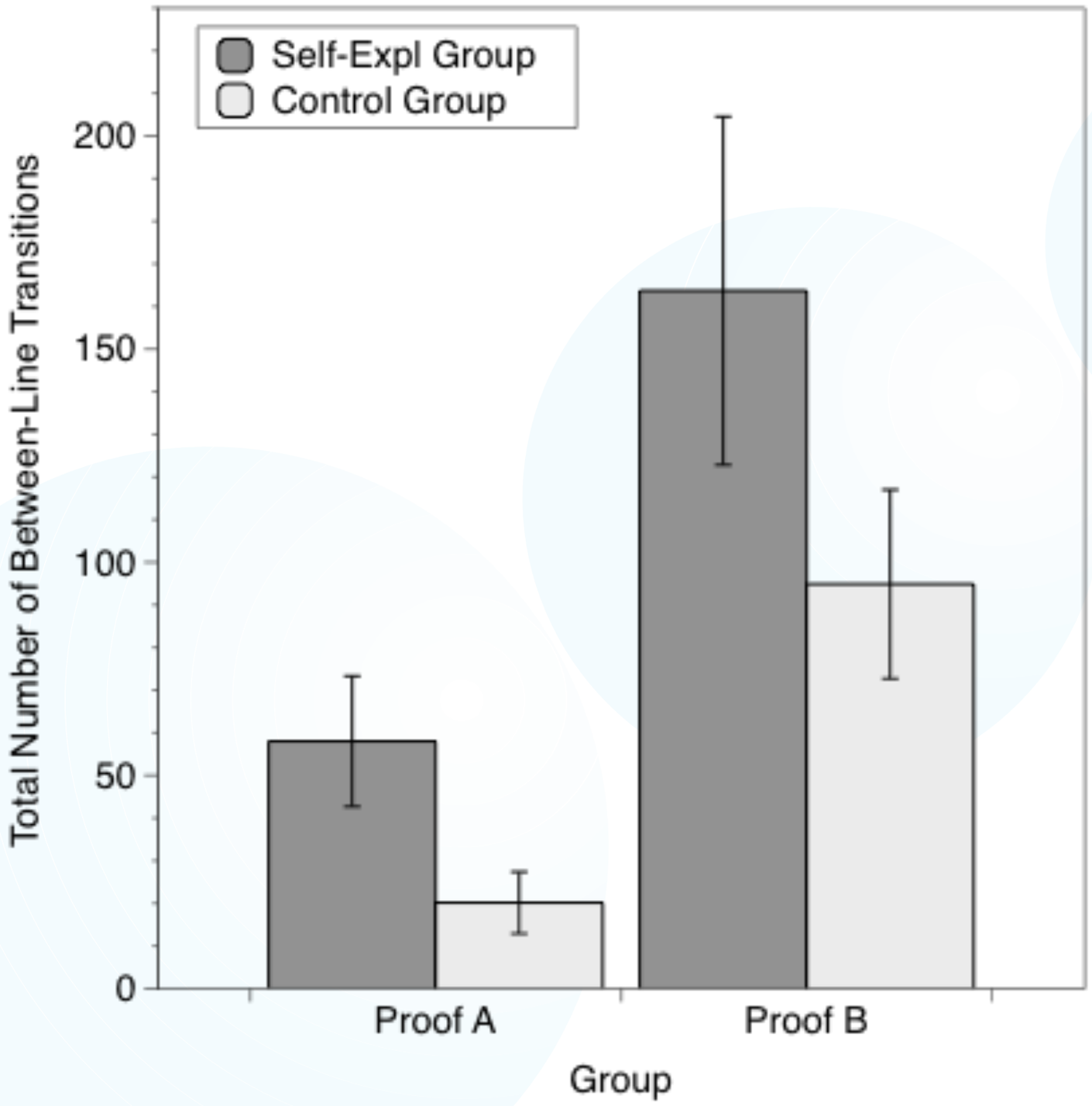

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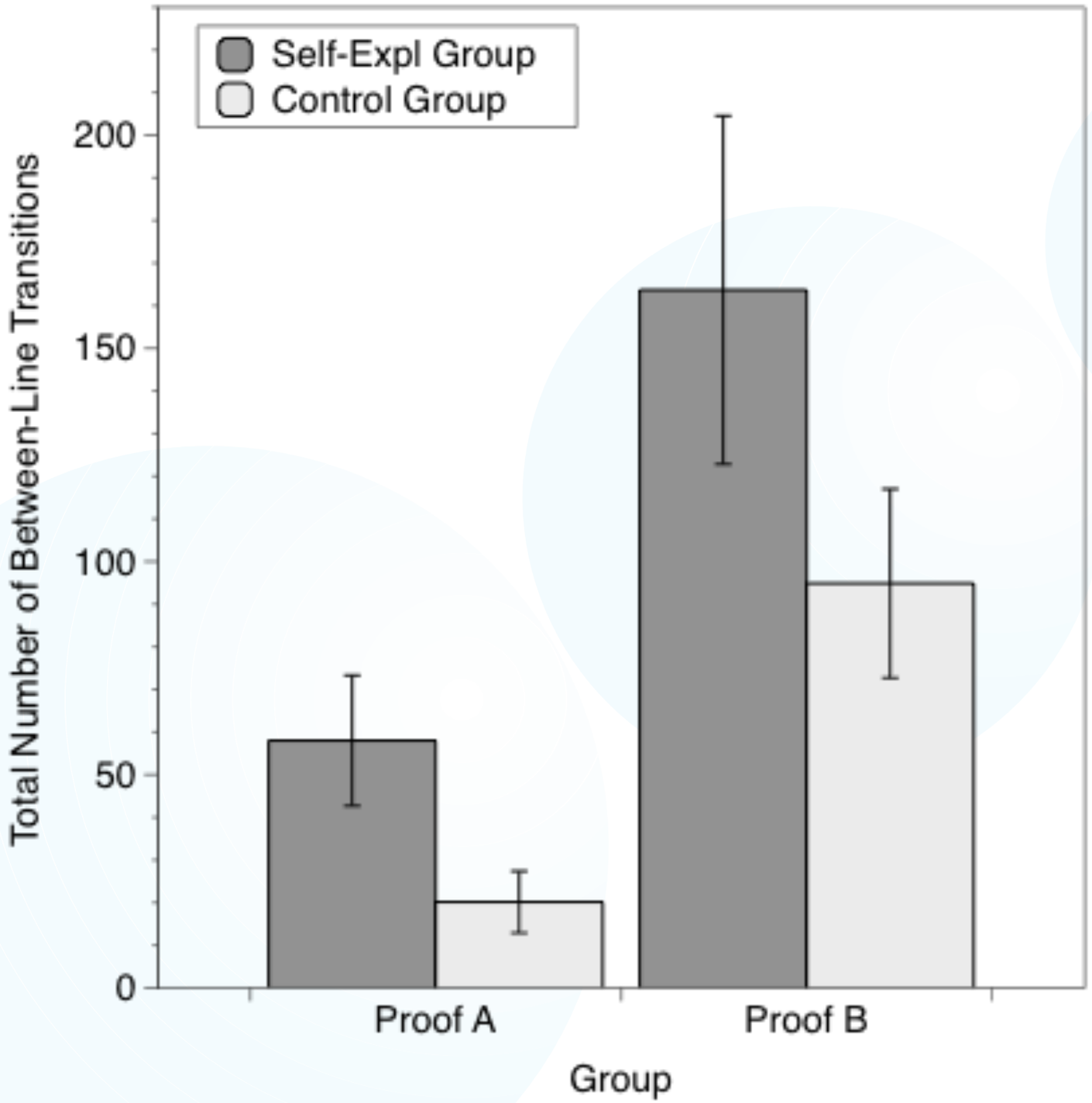

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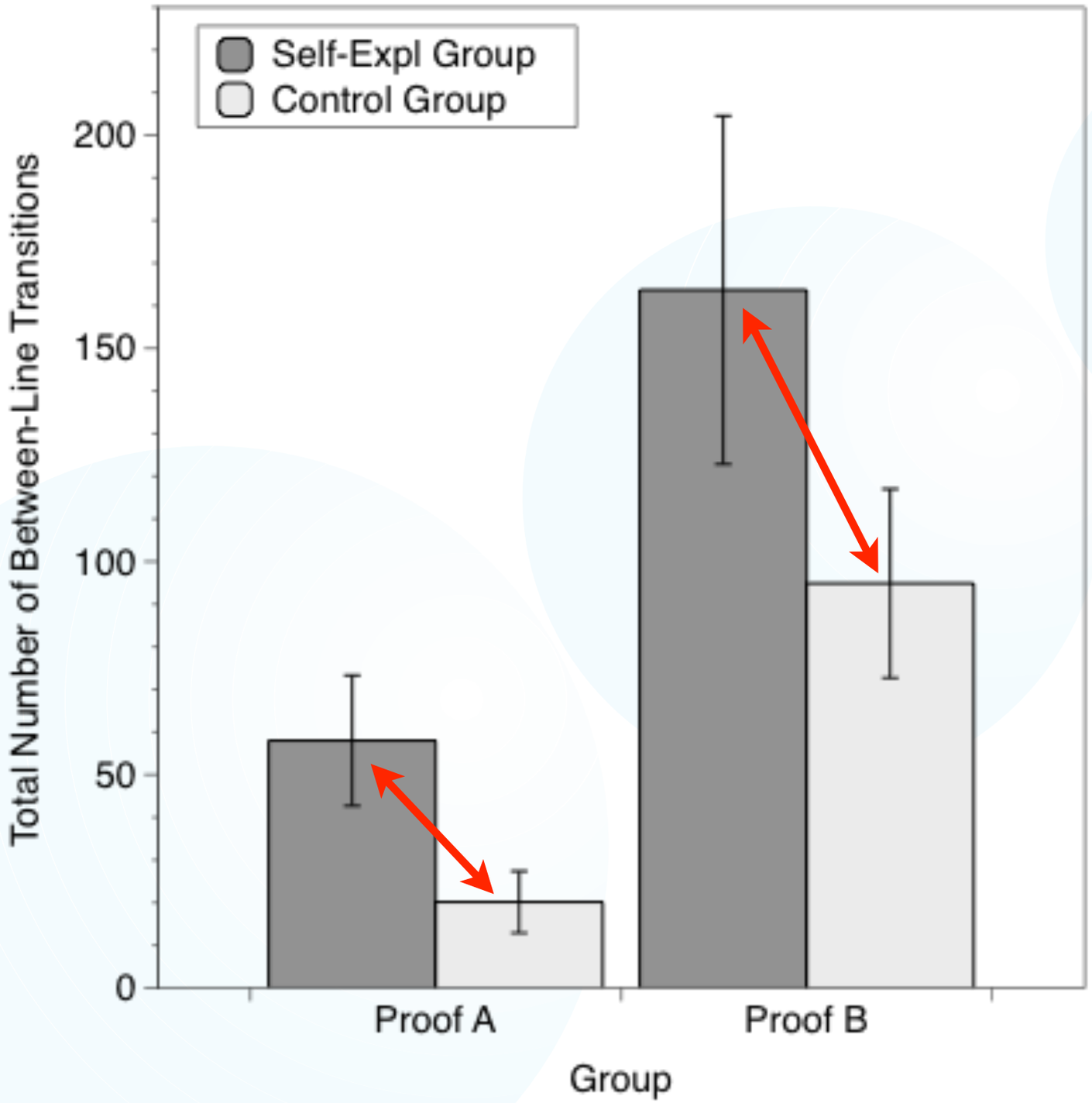

Significant effect of condition (p=.004)

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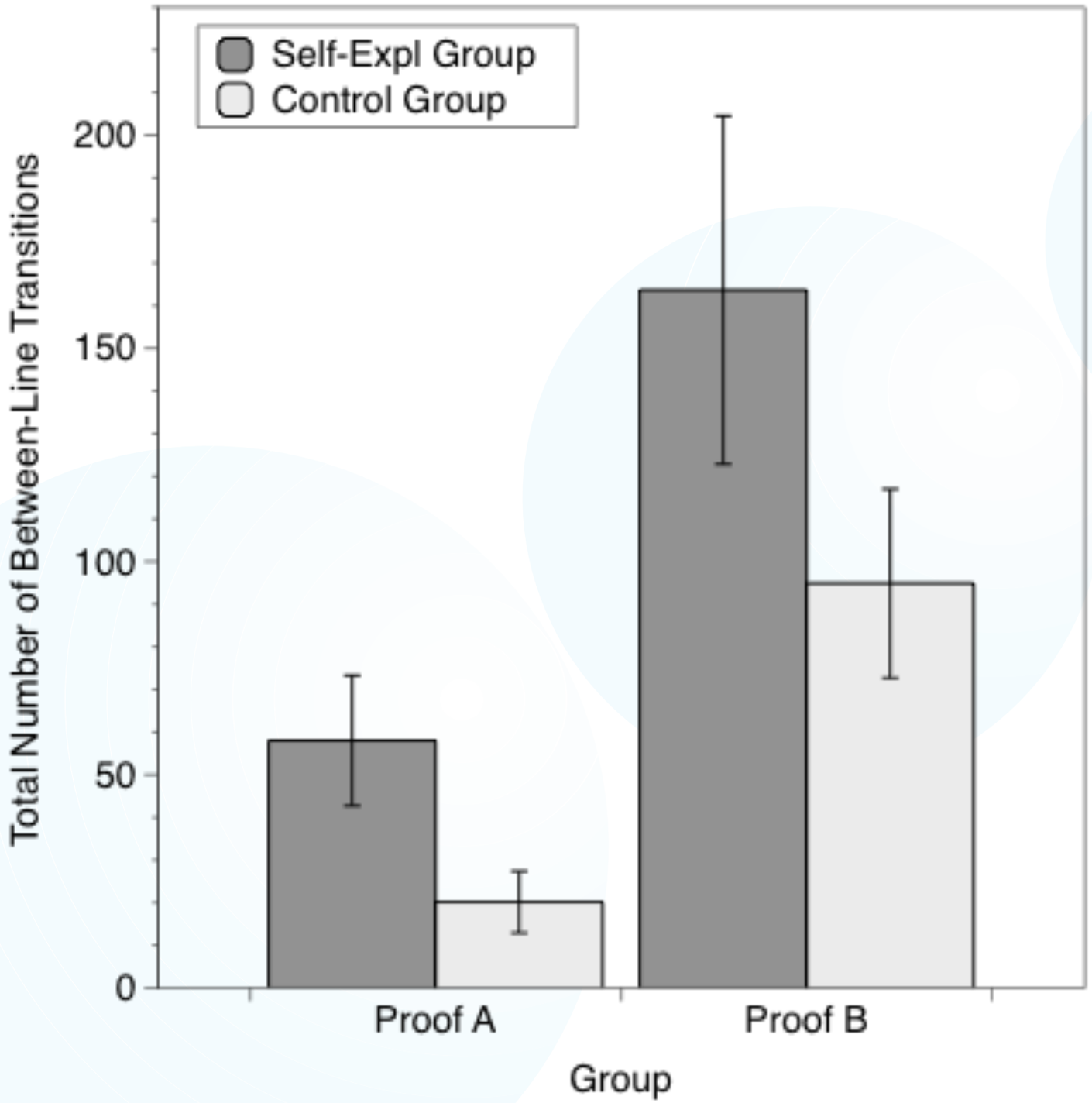

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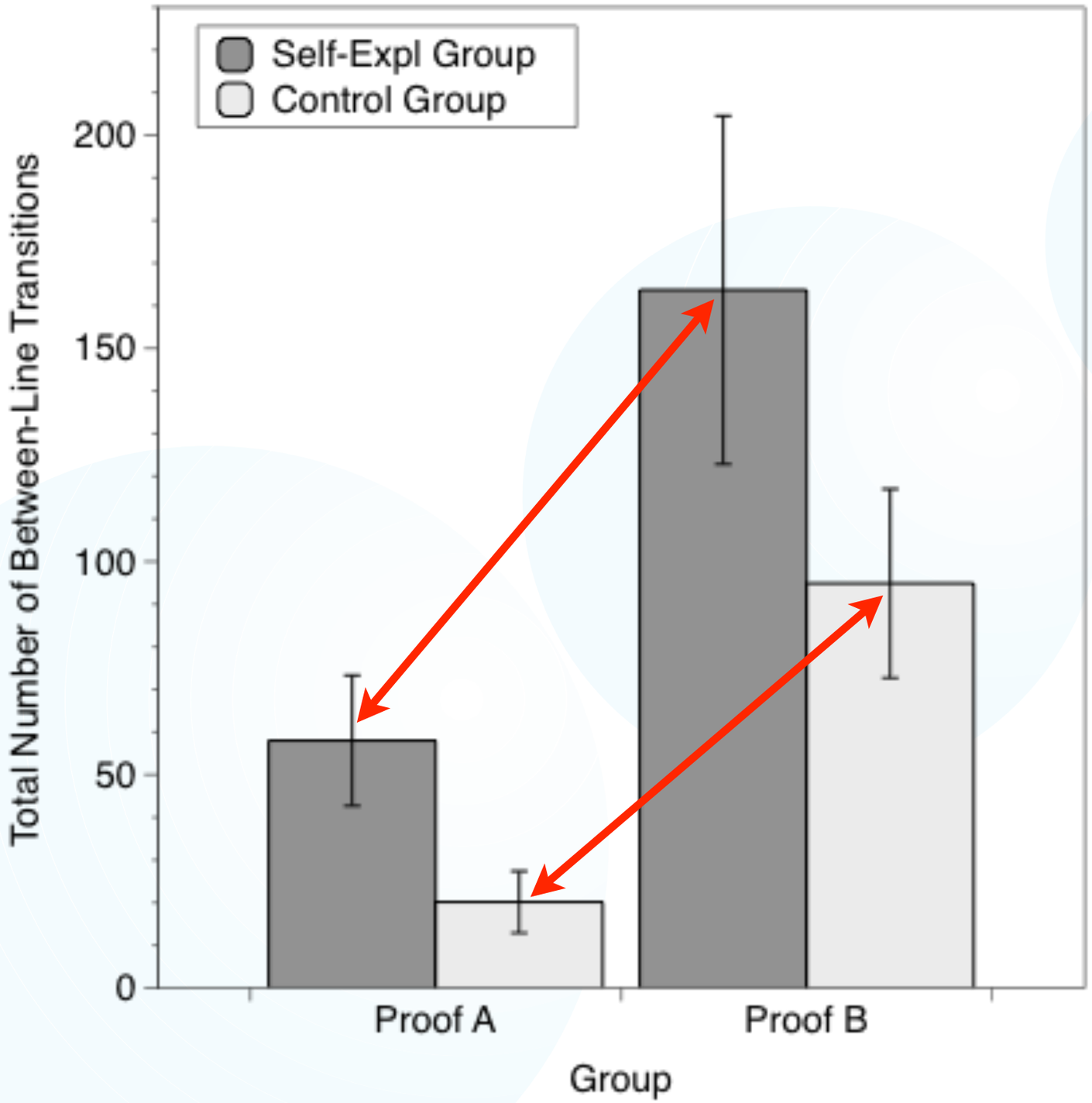

Significant effect of proof (p=.008)

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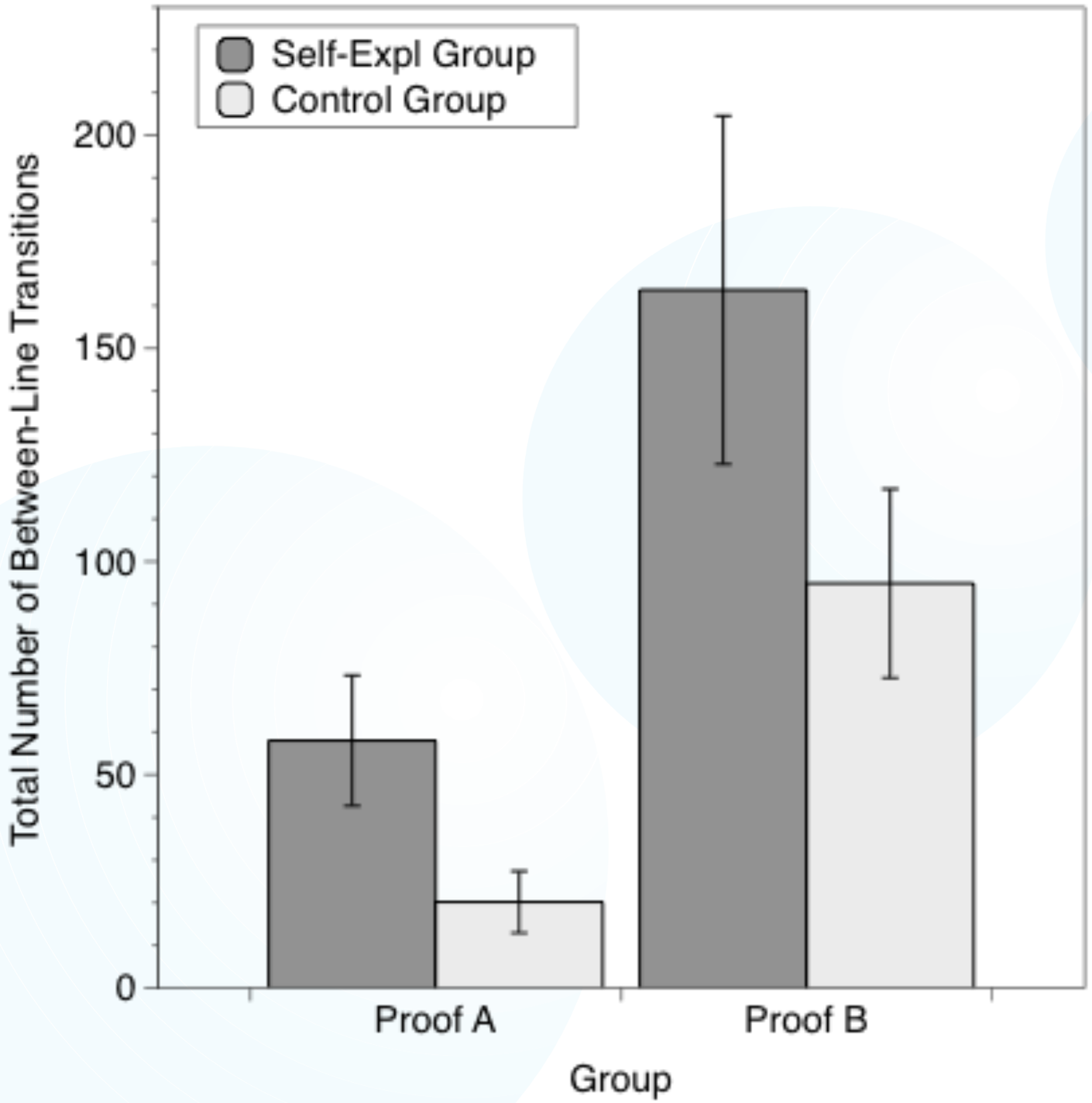

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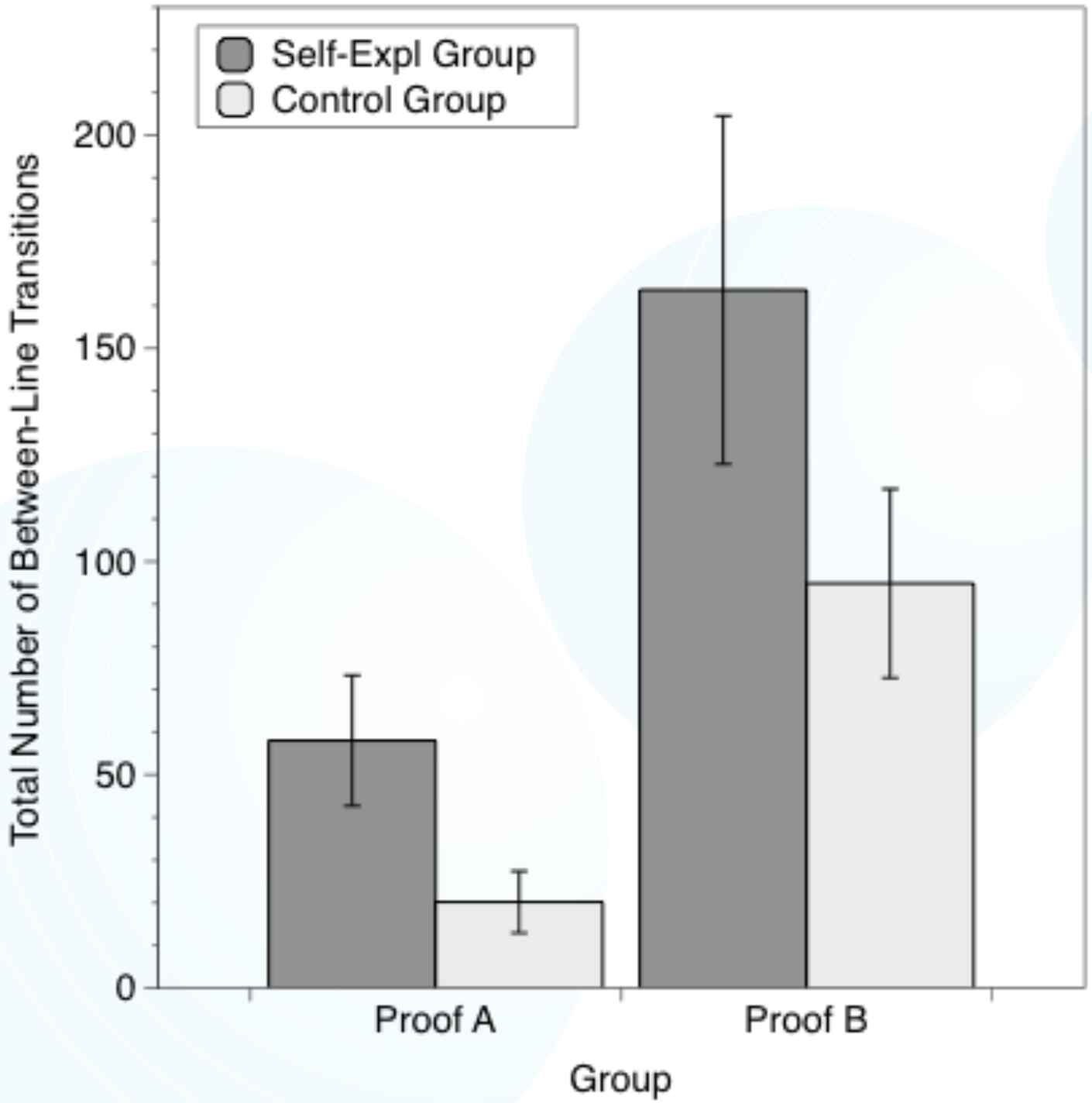

No significant interaction

Self-explanation training does change underlying reading behaviour.

Students concentrate harder and move their attention around more.

Does self-explanation training work in a genuine pedagogical setting?

Method

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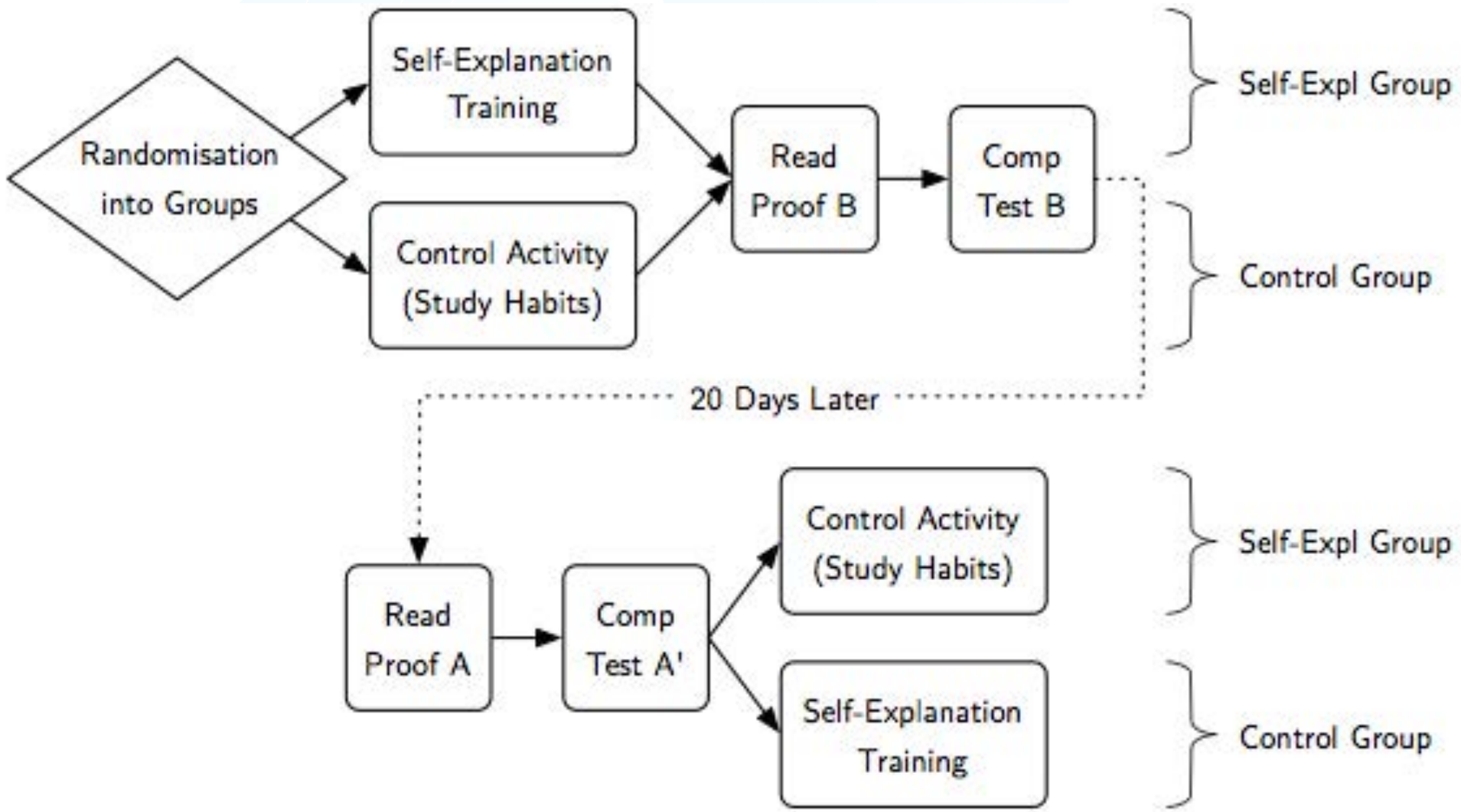

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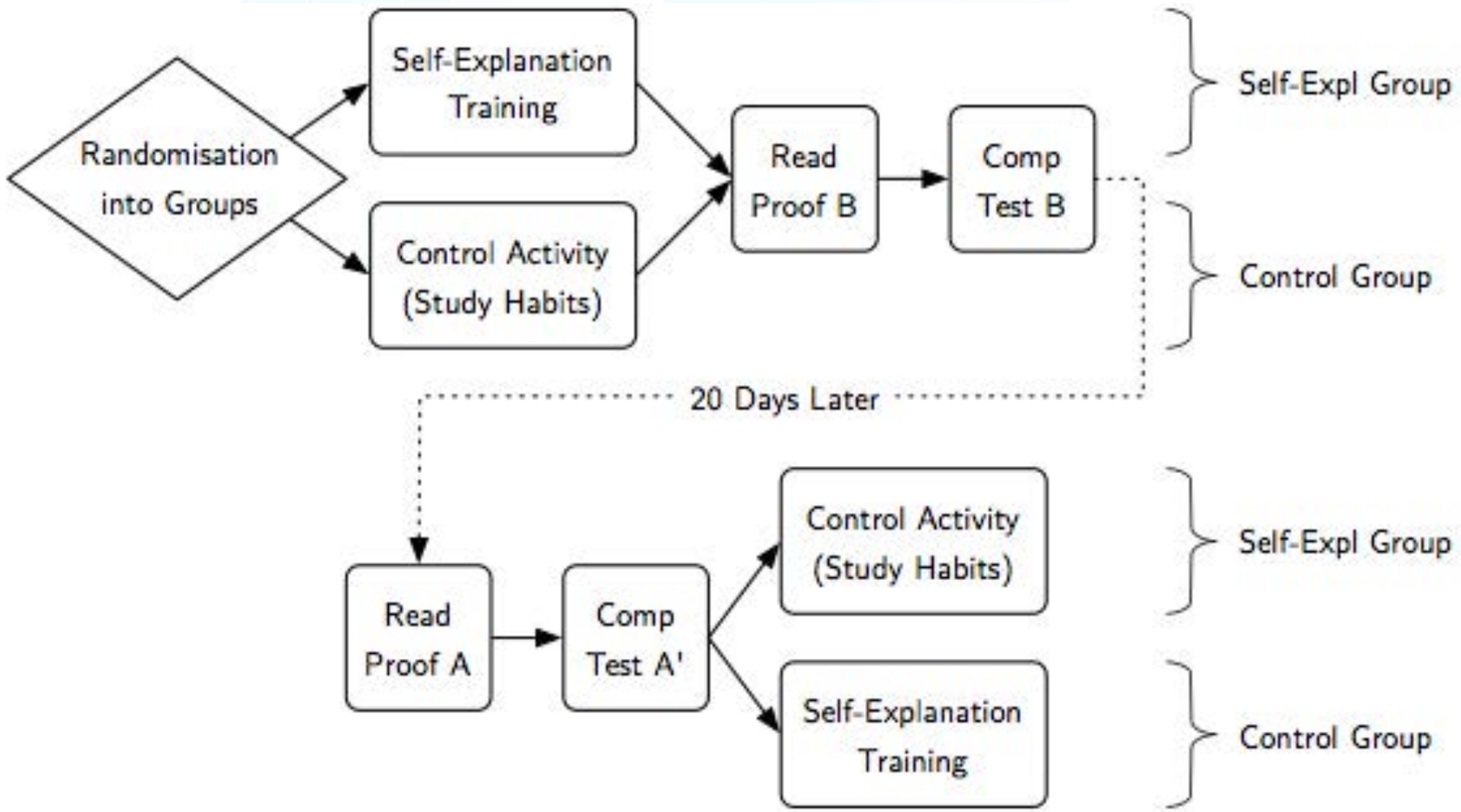
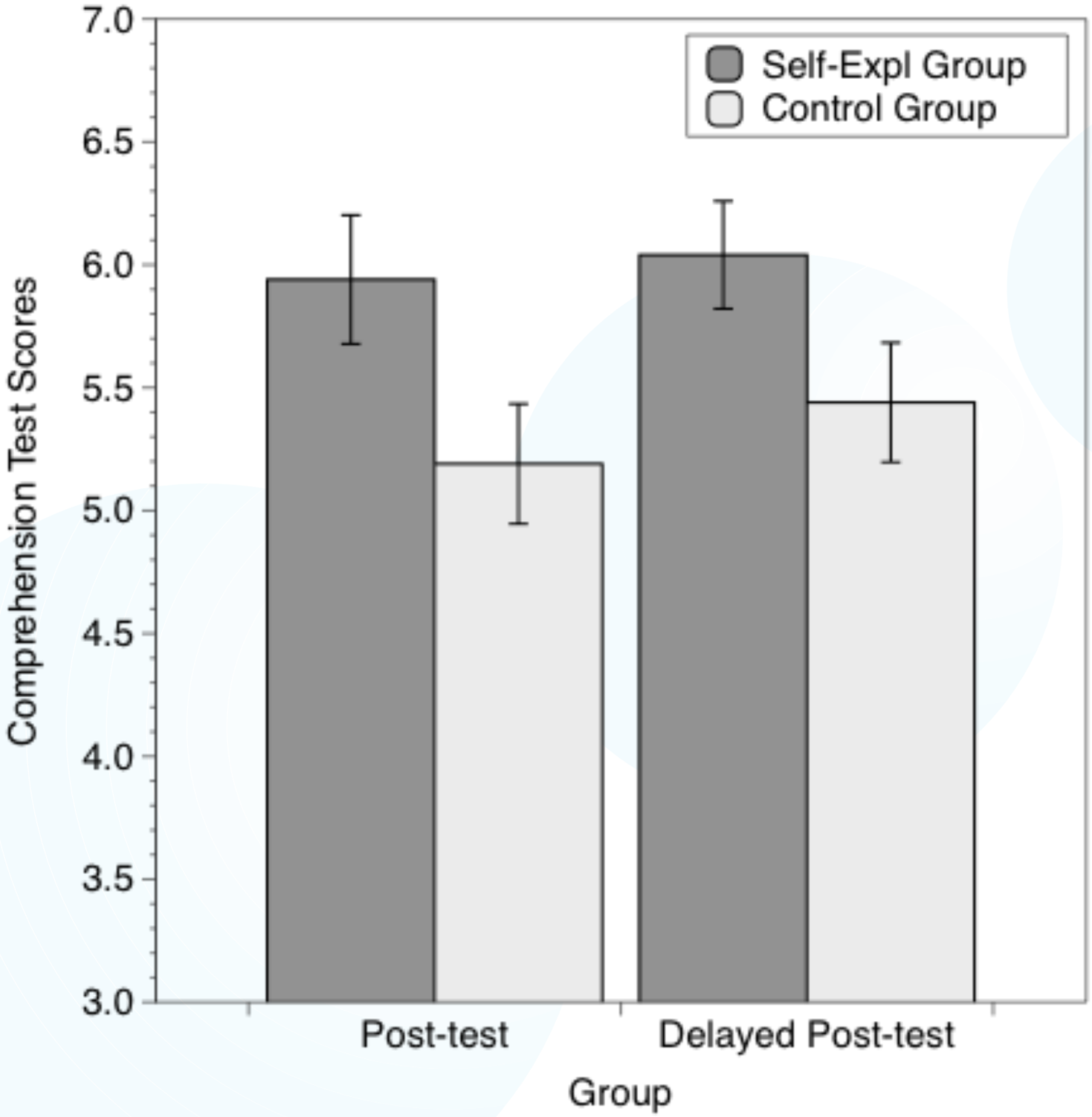

Randomisation into Groups
Self-Explanation Training
Control Activity (Study Habits)
Read Proof B
Comp Test B
Self-Expl Group
Control Group
20 Days Later
Read Proof A
Comp Test A'
Control Activity (Study Habits)
Self-Explanation Training
Self-Expl Group
Control Group

Proof comprehension

Proof comprehension

Measure: proof comprehension scores (out of 10).

Proof comprehension



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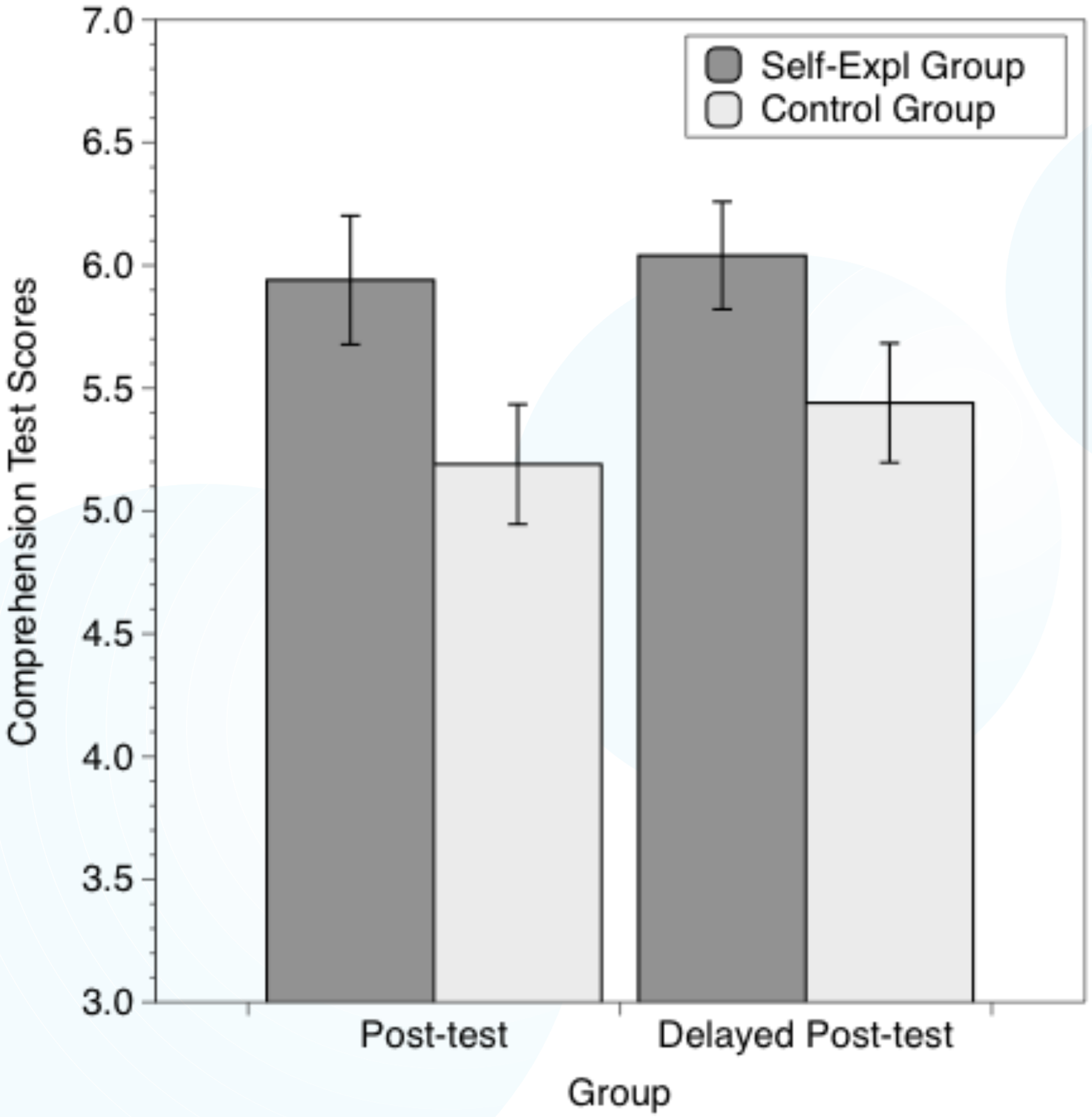

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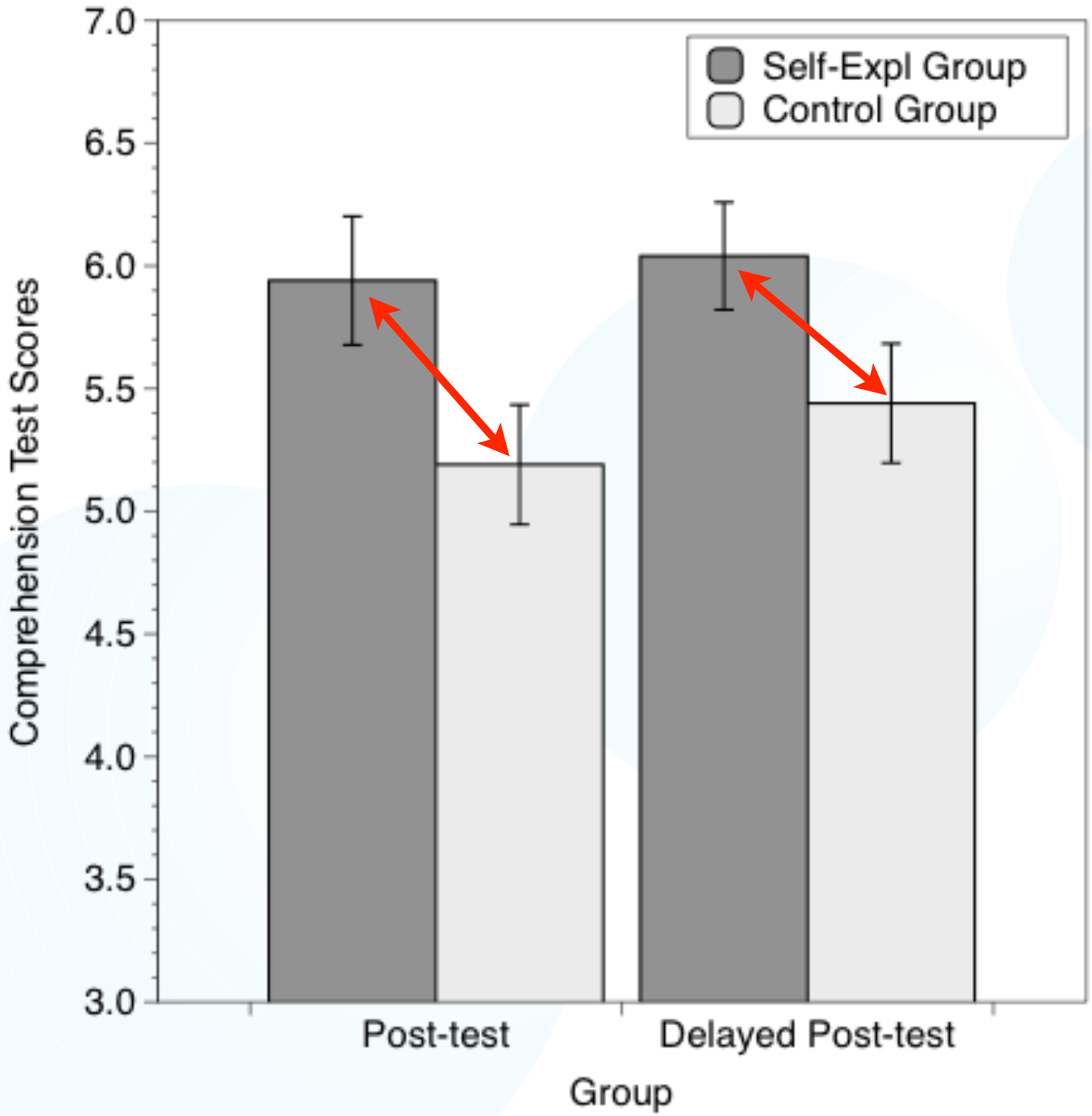
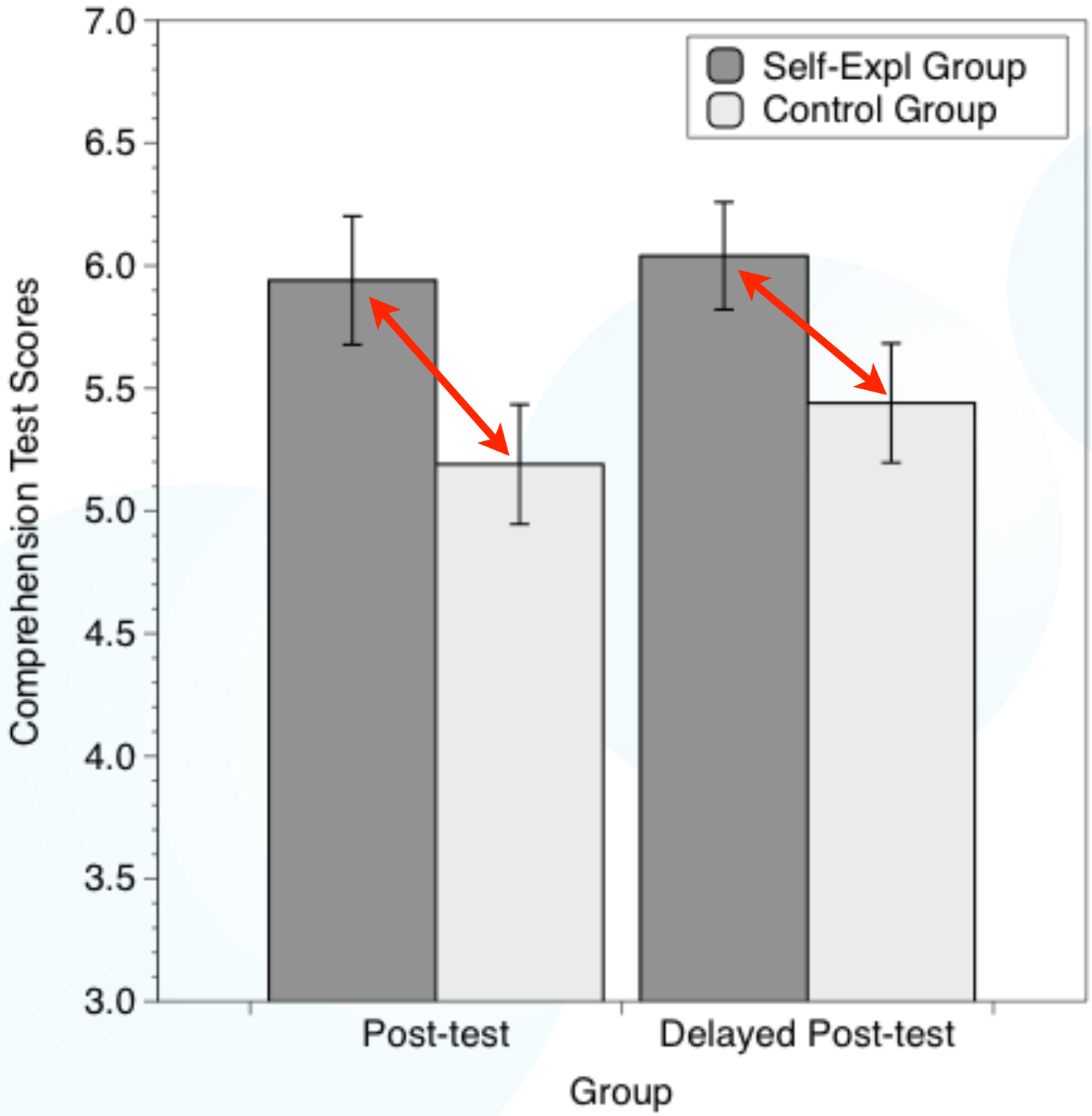


Significant effect of condition (p=.016)

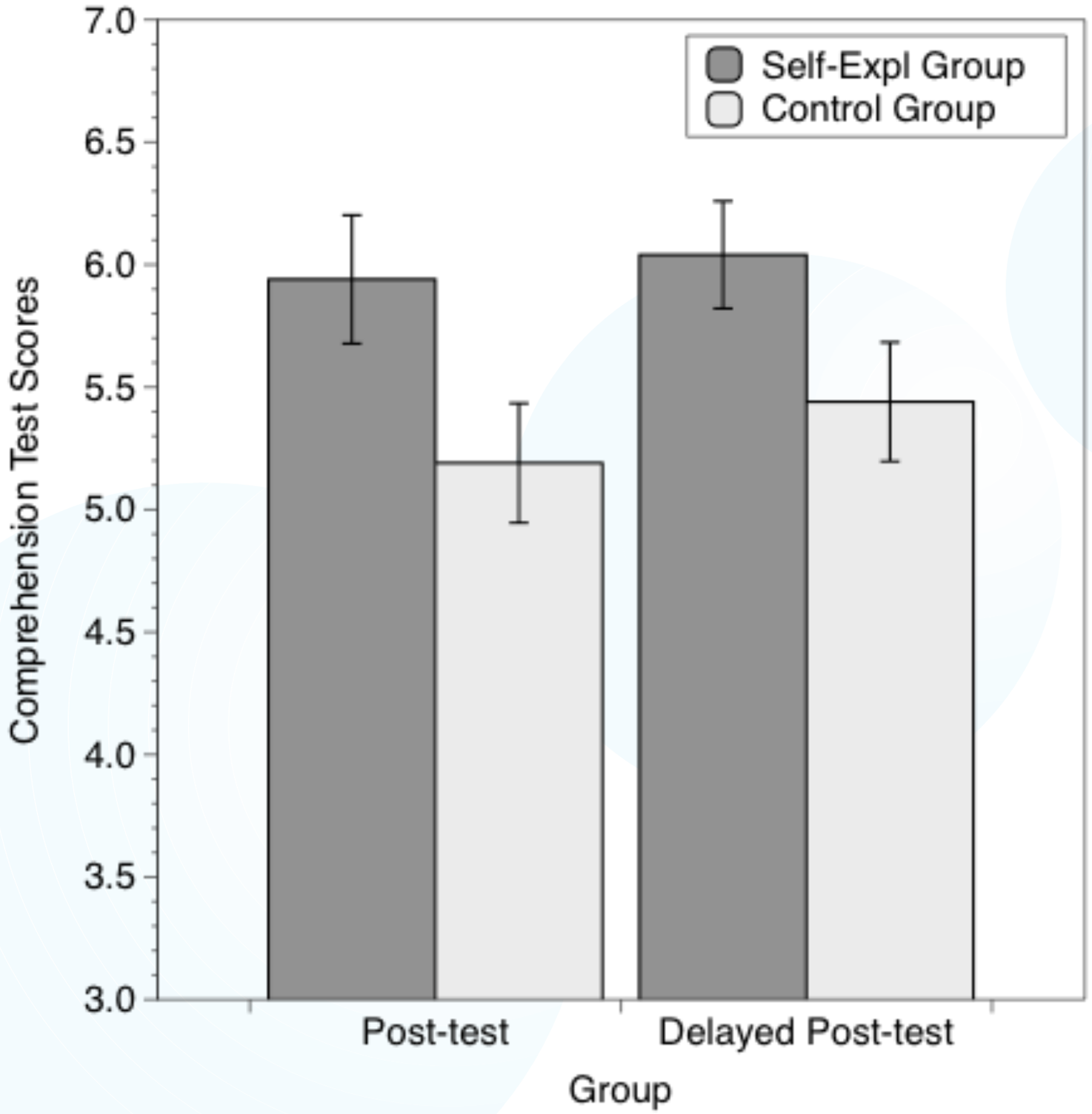
Proof comprehension



Significant effect of condition (p=.016)

No significant effect of time; no significant interaction

Proof comprehension



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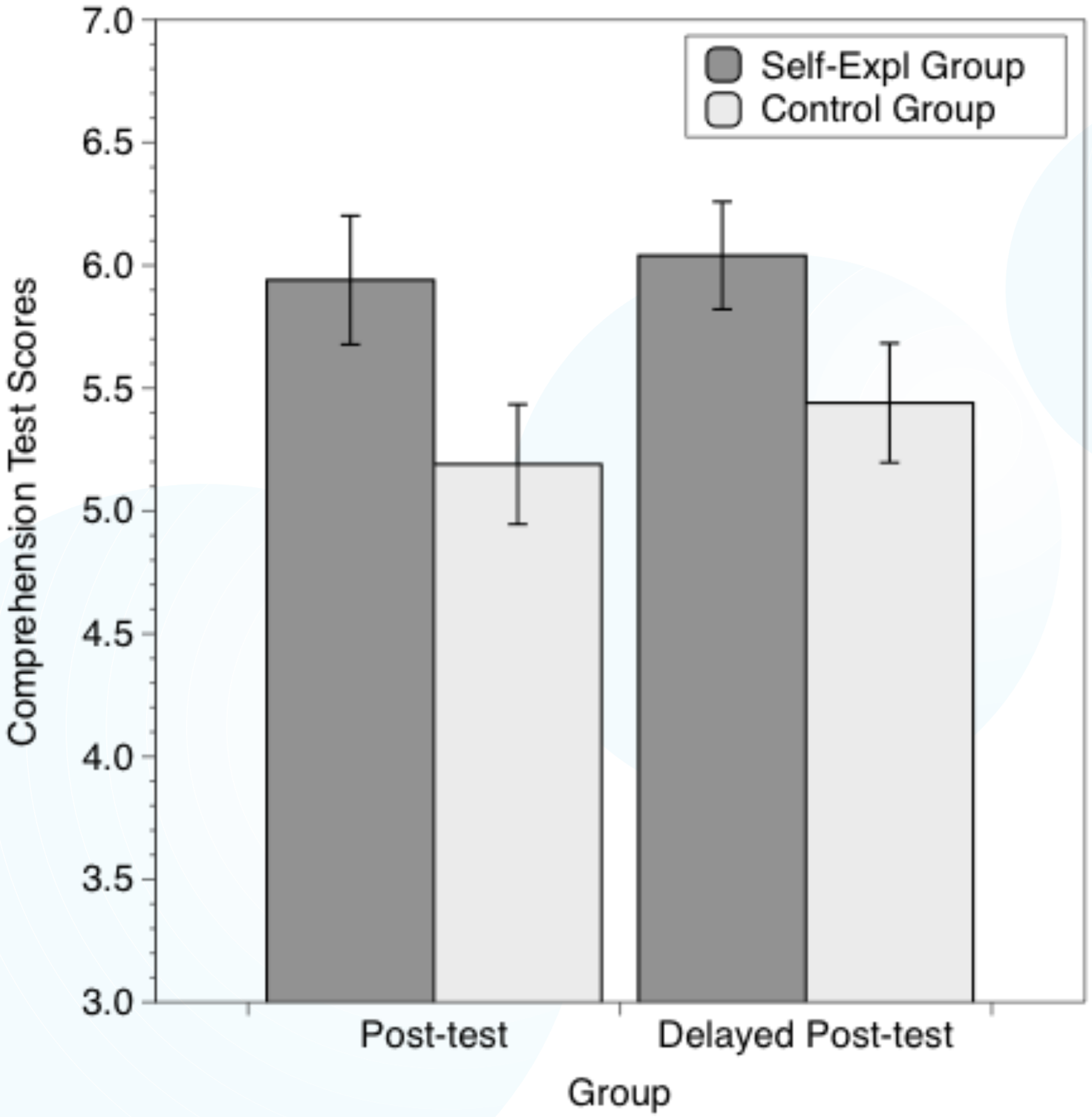
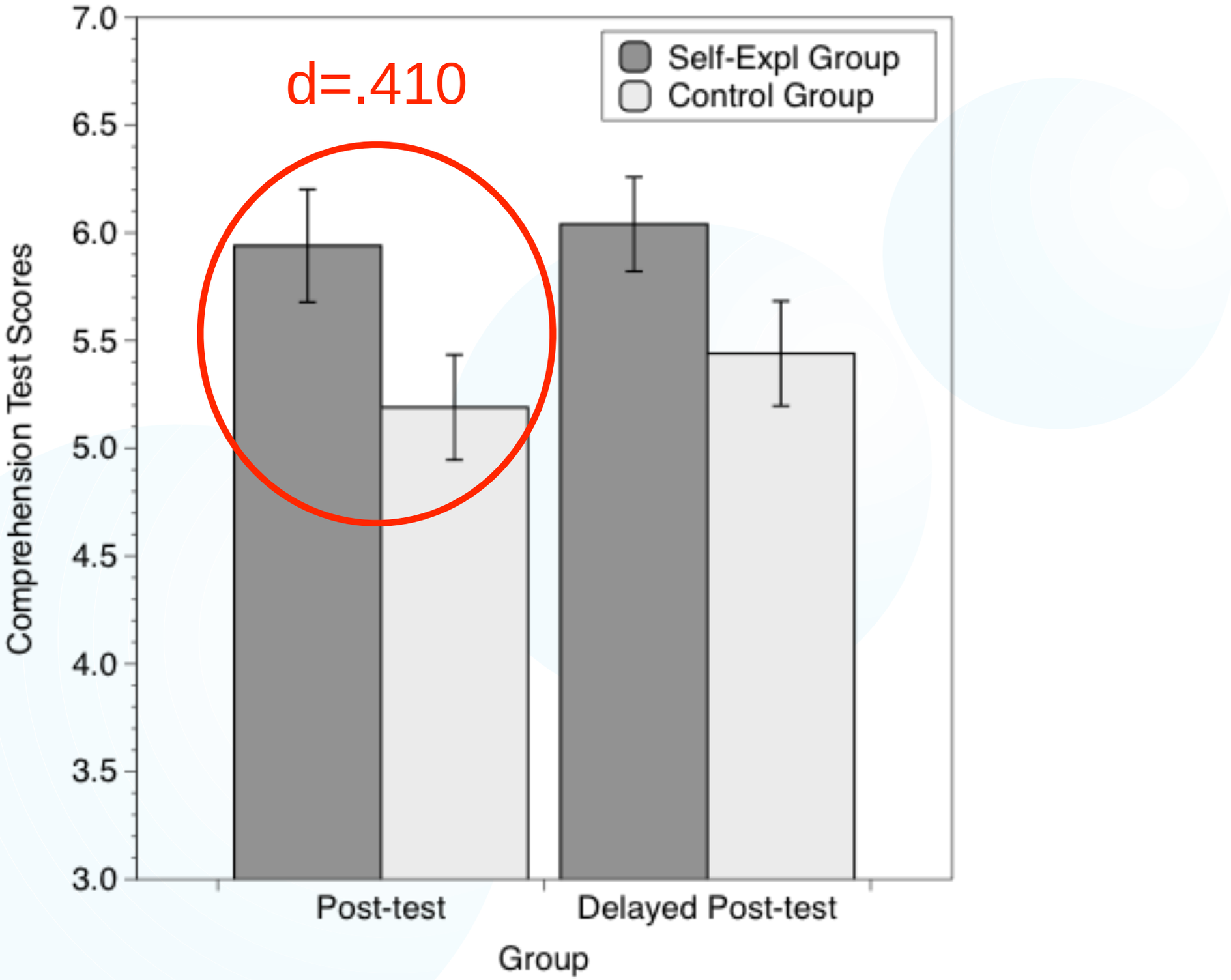
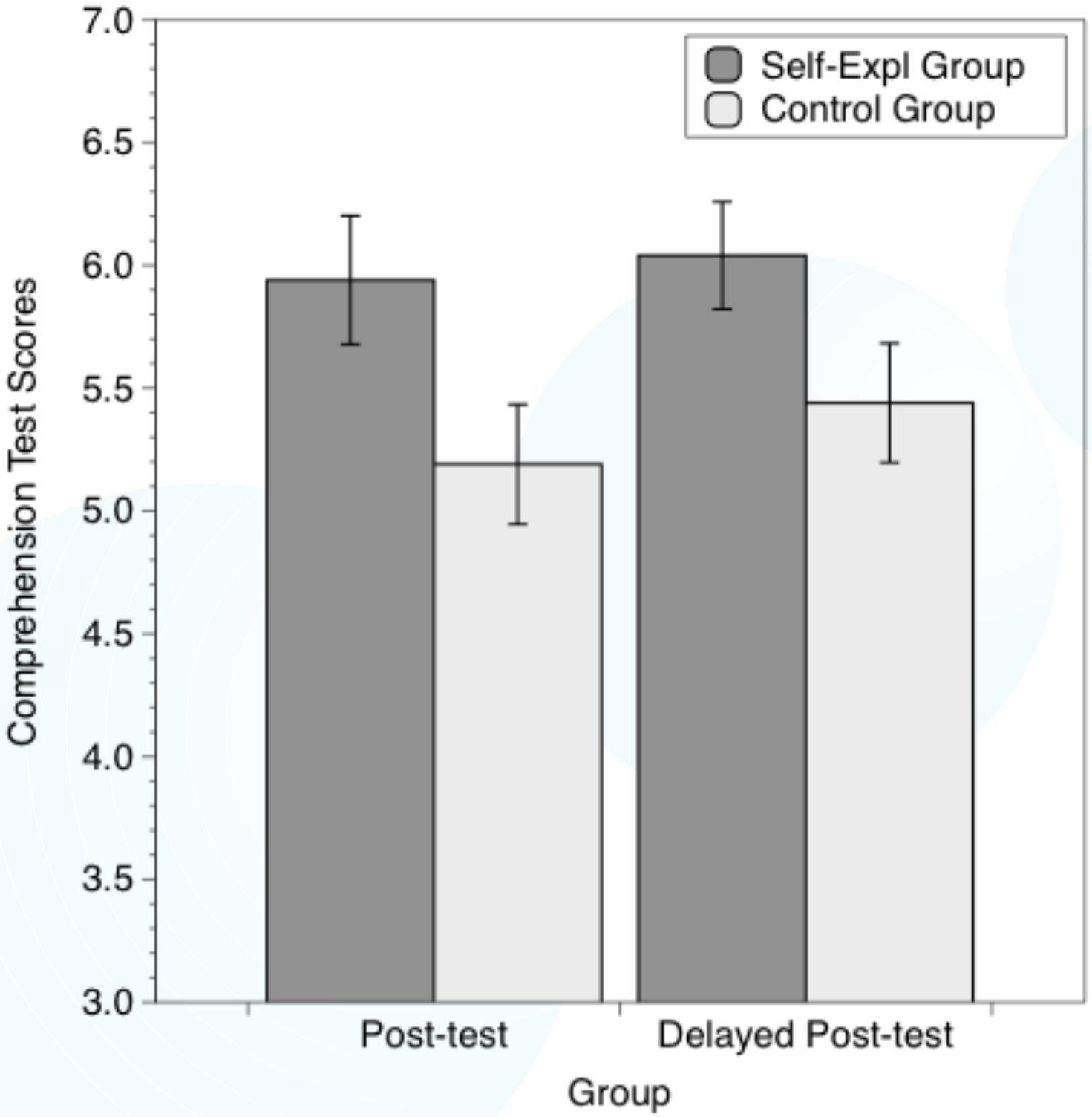
Proof comprehension



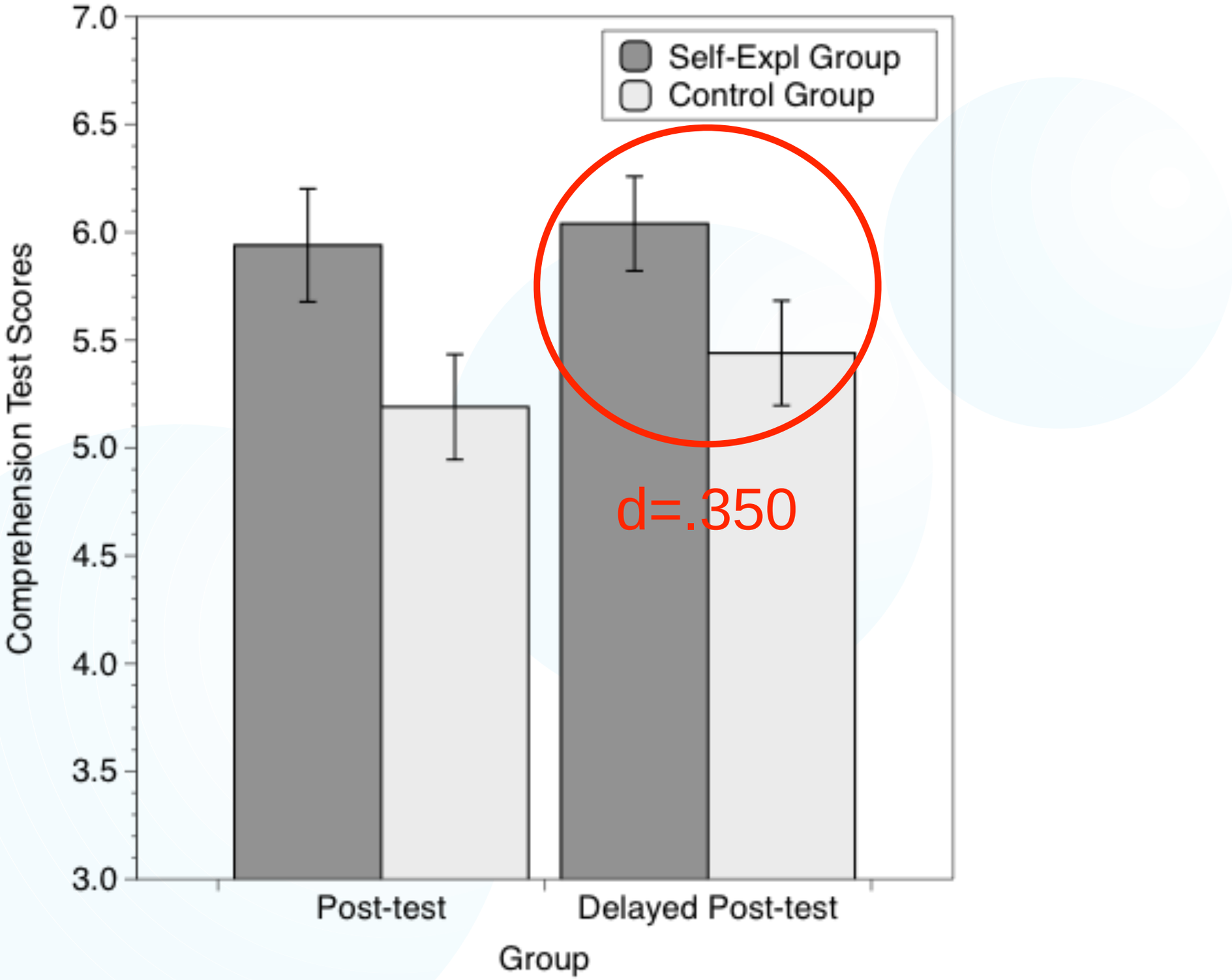
Proof comprehension



Proof comprehension



Proof comprehension

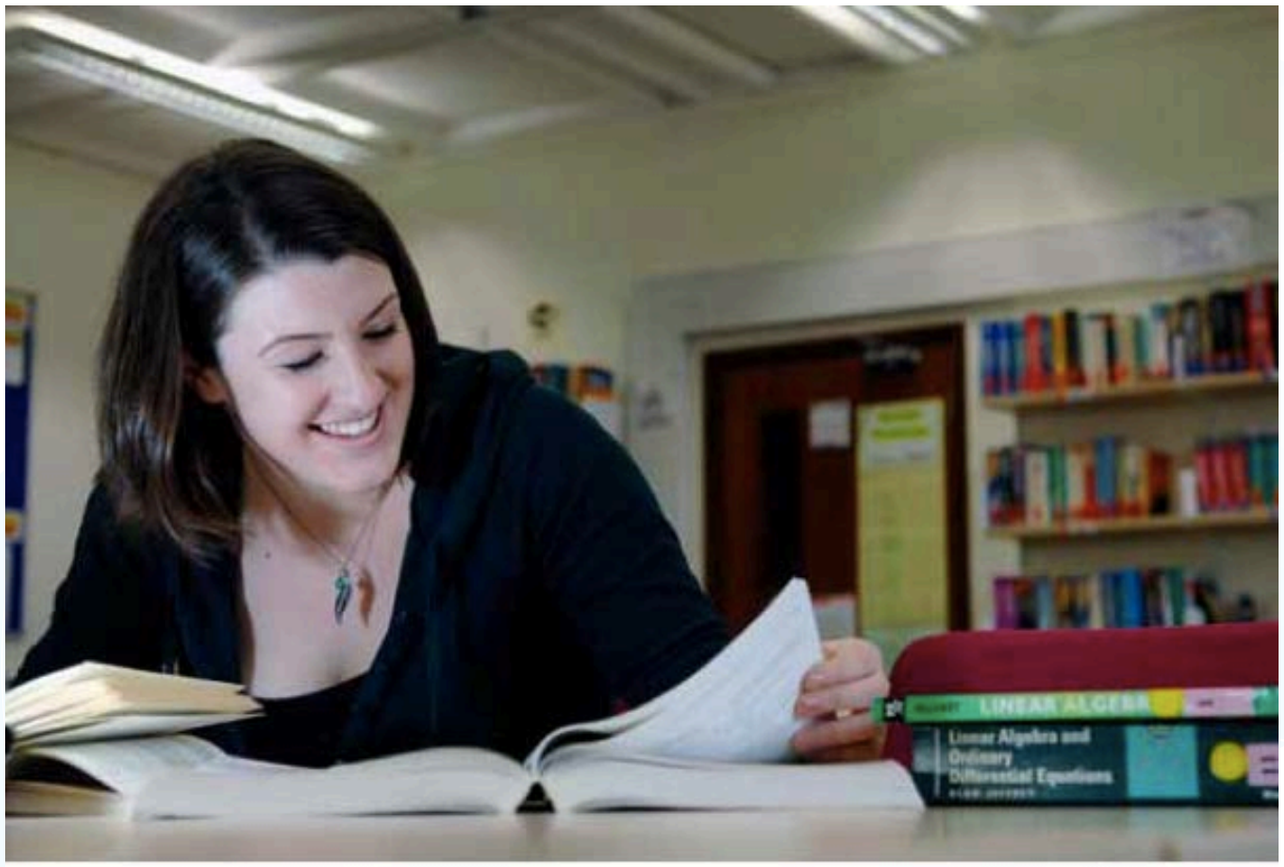


Self-explanation training does work in a genuine pedagogical setting, and the effect lasts.

Thank you.

Thanks to the MSOR Network and to Loughborough University for funding.

Please get in touch if you would like copies of the self-explanation training booklet:
l.j.alcock@lboro.ac.uk



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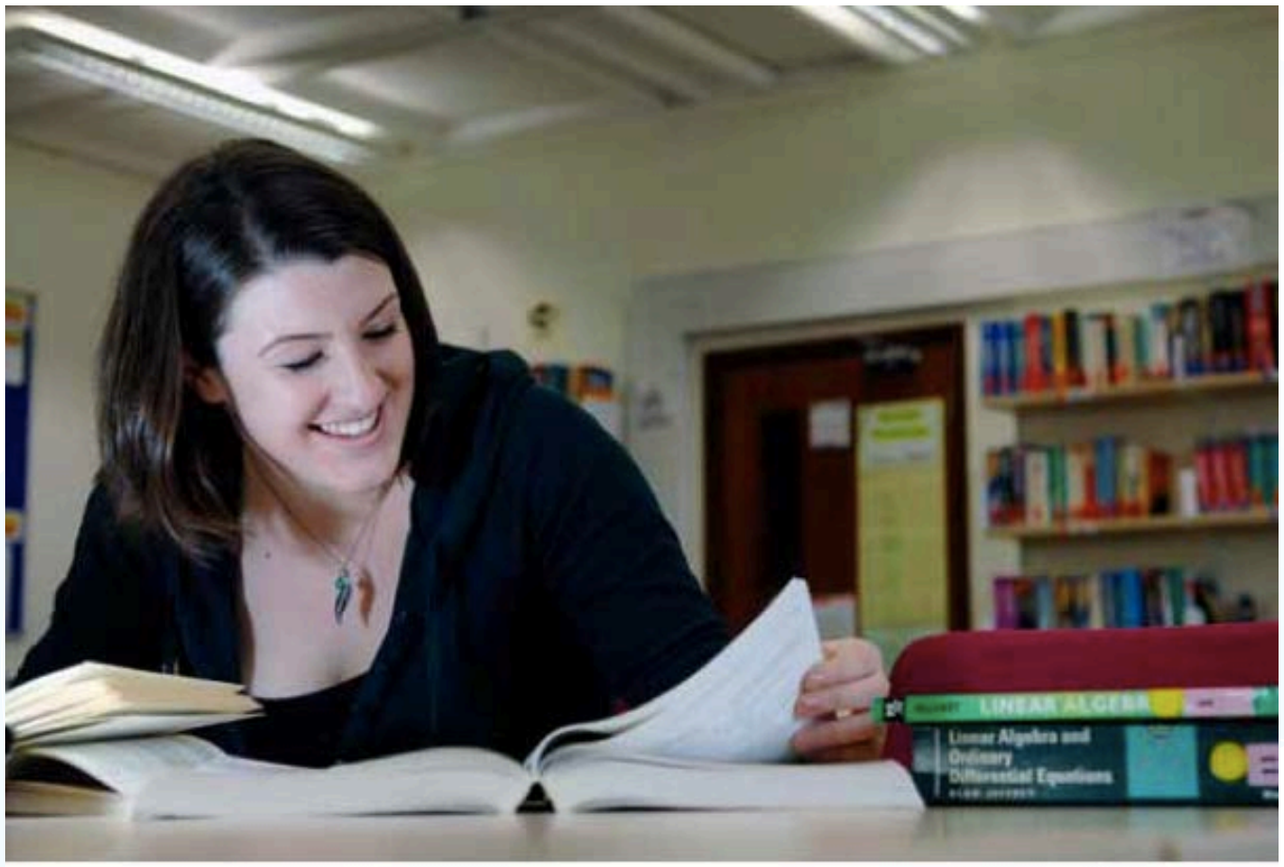

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Inglis, M., & Alcock, L. (2012). Expert and novice approaches to reading mathematical proofs. *Journal for Research in Mathematics Education*, *43*, 358-390.

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